26. Ellipse

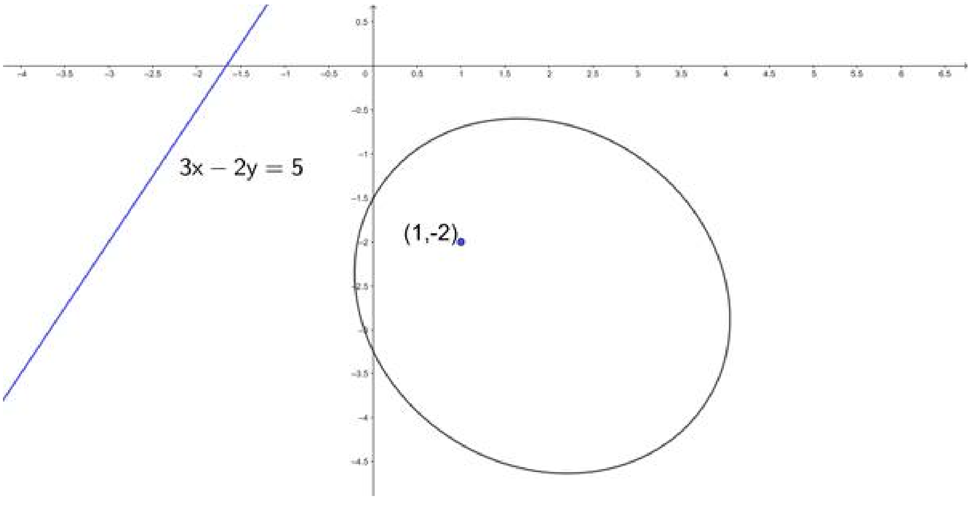
Exercise 26.1

1. Question

Find the equation of the ellipse whose focus is (1, - 2), the directrix 3x – 2y + 5 = 0 and eccentricity equal to 1/2.

Answer

Given that we need to find the equation of the ellipse whose focus is S(1, - 2) and directrix(M) is 3x - 2y + 5 = 0 and eccentricity(e) is equal to $\frac{1}{2}$.



Let P(x,y) be any point on the ellipse.

We know that the distance between the focus and any point on the ellipse is equal to the eccentricity times the perpendicular distance from that point to the directrix.

We know that distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

We know that the perpendicular distance from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$.

⇒ SP = ePM

⇒ $SP^2 = e^2PM^2$

⇒ $(x-1)^2 + (y-(-2))^2 = \left(\frac{1}{2}\right)^2\left(\frac{|3x-2y+5|}{\sqrt{3^2+(-2)^2}}\right)^2$

⇒ $x^2 - 2x + 1 + y^2 + 4y + 4 = \frac{1}{4} \times \frac{(|3x-2y+5|)^2}{9+4}$

⇒ $x^2 + y^2 - 2x + 4y + 5 = \frac{1}{52} \times (9x^2 + 4y^2 + 25 - 12xy - 20y + 30x)$

⇒ $52x^2 + 52y^2 - 104x + 208y + 260 = 9x^2 + 4y^2 - 12xy - 20y + 30x + 25$

⇒ $43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$

∴ The equation of the ellipse is $43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$.

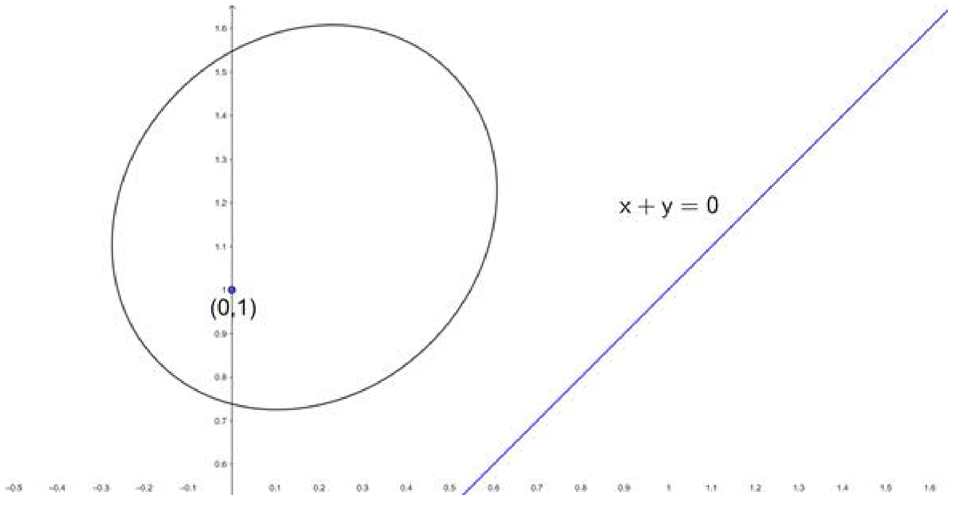
2 A. Question

Find the equation of the ellipse in the following cases:

focus is (0, 1), directrix is x + y = 0 and $e = \frac{1}{2}$.

Answer

Given that we need to find the equation of the ellipse whose focus is S(0,1) and directrix(M) is x + y = 0 and eccentricity(e) is equal to $\frac{1}{2}$.



Let P(x,y) be any point on the ellipse.

We know that the distance between the focus and any point on the ellipse is equal to the eccentricity times the perpendicular distance from that point to the directrix.

We know that distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = ePM

⇒ $SP^2 = e^2PM^2$

⇒ $(x - 0)^2 + (y - 1)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|x + y|}{\sqrt{1^2 + 1^2}}\right)^2$

⇒ $x^2 + y^2 - 2y + 1 = \frac{1}{4} \times \frac{(|x + y|)^2}{1 + 1}$

⇒ $x^2 + y^2 - 2y + 1 = \frac{1}{8} \times (x^2 + y^2 + 2xy)$

⇒ $8x^2 + 8y^2 - 16y + 8 = x^2 + y^2 + 2xy$

⇒ $7x^2 + 7y^2 - 2xy - 16y + 8 = 0$

∴ The equation of the ellipse is $7x^2 + 7y^2 - 2xy - 16y + 8 = 0$.

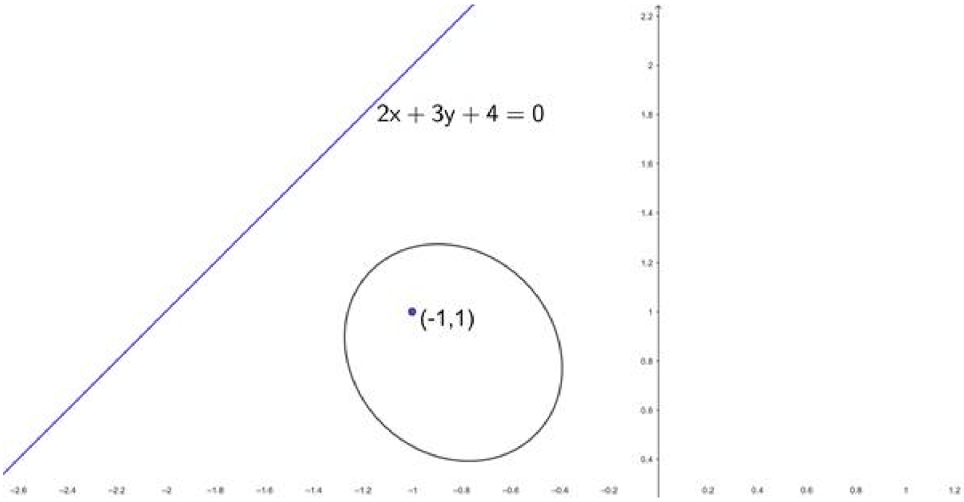
2 B. Question

Find the equation of the ellipse in the following cases:

focus is (- 1, 1), directrix is x - y + 3 = 0 and $e = \frac{1}{2}$.

Answer

Given that we need to find the equation of the ellipse whose focus is S(- 1,1) and directrix(M) is x - y + 3 = 0 and eccentricity(e) is equal to $\frac{1}{2}$.



Let P(x,y) be any point on the ellipse.

We know that the distance between the focus and any point on the ellipse is equal to the eccentricity times the perpendicular distance from that point to the directrix.

We know that distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = ePM

⇒ $SP^2 = e^2PM^2$

⇒ $(x - (-1))^2 + (y - 1)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|x-y+3|}{\sqrt{1^2 + 1^2}}\right)^2$

⇒ $x^2 + 2x + 1 + y^2 - 2y + 1 = \frac{1}{4} \times \frac{(|x-y+3|)^2}{1+1}$

⇒ $x^2 + y^2 + 2x - 2y + 2 = \frac{1}{8} \times (x^2 + y^2 + 9 - 2xy - 6y + 6x)$

⇒ $8x^2 + 8y^2 + 16x - 16y + 16 = x^2 + y^2 - 2xy + 6x - 6y + 9$

⇒ $7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$

∴ The equation of the ellipse is $7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$.

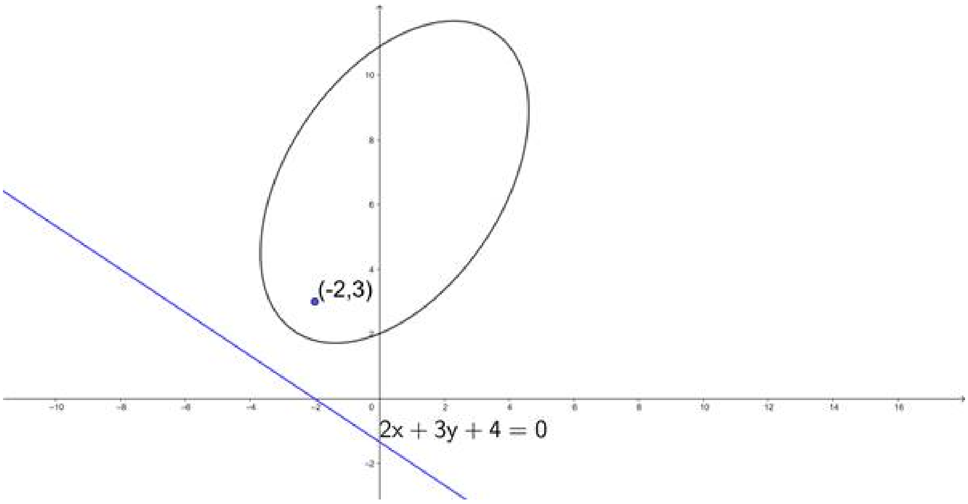
2 C. Question

Find the equation of the ellipse in the following cases:

focus is (- 2, 3), directrix is 2x + 3y + 4 = 0 and $e = \frac{4}{5}$

Answer

Given that we need to find the equation of the ellipse whose focus is S(- 2,3) and directrix(M) is 2x + 3y + 4 = 0 and eccentricity(e) is equal to $\frac{4}{5}$.



Let P(x,y) be any point on the ellipse.

We know that the distance between the focus and any point on the ellipse is equal to the eccentricity times the perpendicular distance from that point to the directrix.

We know that distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

$\Rightarrow$ SP = ePM

$\Rightarrow SP^2 = e^2PM^2$

$$\Rightarrow (x - (-2))^2 + (y - 3)^2 = \left(\frac{4}{5}\right)^2 \left(\frac{|2x + 3y + 4|}{\sqrt{2^2 + 3^2}}\right)^2$$

$$\Rightarrow x^2 + 4x + 4 + y^2 - 6y + 9 = \frac{16}{25} \times \frac{(|2x + 3y + 4|)^2}{4 + 9}$$

$$\Rightarrow x^2 + y^2 + 4x - 6y + 13 = \frac{16}{325} \times (4x^2 + 9y^2 + 16 + 12xy + 16x + 24y)$$

$\Rightarrow 325x^2 + 325y^2 + 1300x - 1950y + 4225 = 64x^2 + 144y^2 + 192xy + 256x + 384y + 256$

$\Rightarrow 261x^2 + 181y^2 - 192xy + 1044x - 2334y + 3969 = 0$

$\therefore$ The equation of the ellipse is $261x^2 + 181y^2 - 192xy + 1044x - 2334y + 3969 = 0$.

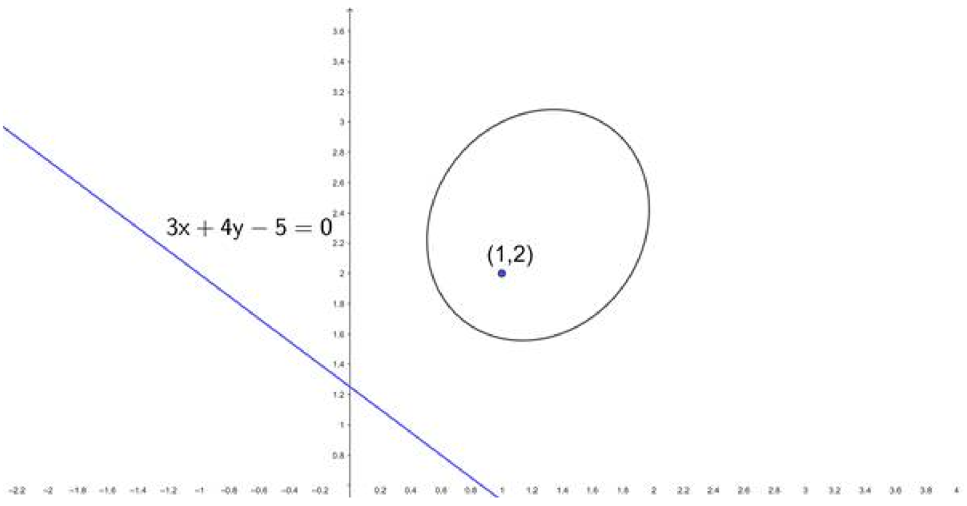
2 D. Question

Find the equation of the ellipse in the following cases:

focus is (1, 2), directrix is 3x + 4y - 7 = 0 and $e = \frac{1}{2}$.

Answer

Given that we need to find the equation of the ellipse whose focus is S(1, 2) and directrix(M) is 3x + 4y - 5 = 0 and eccentricity(e) is equal to $\frac{1}{2}$.



Let P(x,y) be any point on the ellipse.

We know that the distance between the focus and any point on the ellipse is equal to the eccentricity times the perpendicular distance from that point to the directrix.

We know that distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

We know that the perpendicular distance from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$.

$\Rightarrow SP = ePM$

$\Rightarrow SP^2 = e^2PM^2$

$\Rightarrow (x-1)^2+(y-2)^2 = \left(\frac{1}{2}\right)^2\left(\frac{|3x+4y-5|}{\sqrt{3^2+4^2}}\right)^2$

$\Rightarrow x^2-2x+1+y^2-4y+4 = \frac{1}{4}\times\frac{(|3x+4y-5|)^2}{9+16}$

$\Rightarrow x^2+y^2-2x-4y+5 = \frac{1}{100}\times(9x^2+16y^2+25+24xy-30x-40y)$

$\Rightarrow 100x^2 + 100y^2 - 200x - 400y + 500 = 9x^2 + 16y^2 + 24xy - 30x - 40y + 25$

$\Rightarrow 91x^2 + 84y^2 - 24xy - 170x - 360y + 475 = 0$

∴ The equation of the ellipse is $91x^2 + 84y^2 - 24xy - 170x - 360y + 475 = 0$.

3 A. Question

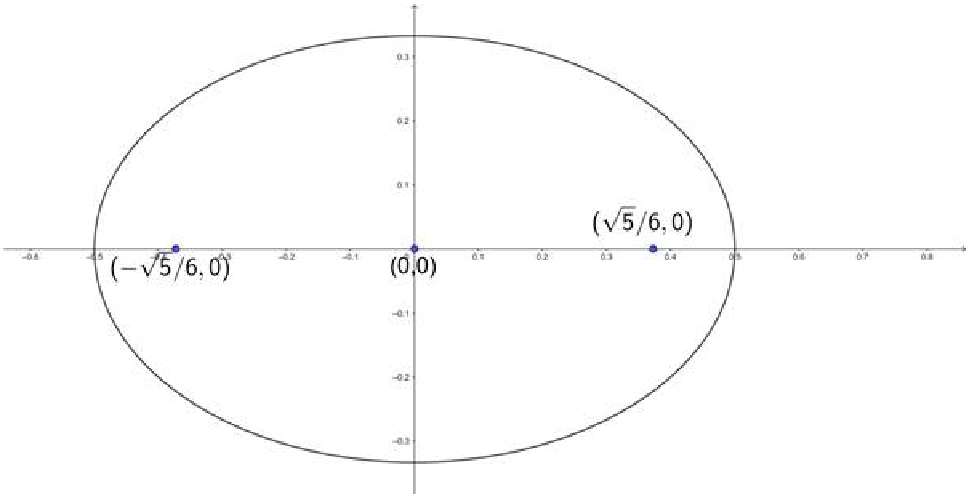
Find the eccentricity, coordinates of foci, length of the latus - rectum of the following ellipse:

$4x^2 + 9y^2 = 1$

Answer

Given the equation of the ellipse is $4x^2 + 9y^2 = 1$.

We need to find the eccentricity, coordinates of foci and length of latus rectum.



Given equation can be rewritten as $\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$.

We know for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$($a^2$>$b^2$)

$\Rightarrow e = \sqrt{\frac{a^2-b^2}{a^2}}$

⇒ Coordinates of foci (±ae,0)

⇒ Length of latus rectum $= \frac{2b^2}{a}$

Here $a^2 = \frac{1}{4}$ and $b^2 = \frac{1}{9}$, a^2>b^2

$\Rightarrow e = \sqrt{\frac{\frac{1}{4}-\frac{1}{9}}{\frac{1}{4}}}$

$\Rightarrow e = \sqrt{\frac{\frac{5}{36}}{\frac{1}{4}}}$

$\Rightarrow e = \sqrt{\frac{5}{9}}$

$\Rightarrow e = \frac{\sqrt{5}}{3}$

$\Rightarrow \text{foci} = \left(\pm\frac{1}{2}\times\frac{\sqrt{5}}{3}, 0\right)$

$\Rightarrow \text{foci} = \left(\pm\frac{\sqrt{5}}{6}, 0\right)$

⇒ Length of latus rectum (L) $= \frac{2\left(\frac{1}{9}\right)}{\frac{1}{2}}$

$\Rightarrow L = \frac{4}{9}$

∴ The eccentricity is $\frac{\sqrt{5}}{3}$, foci are $\left(\pm\frac{\sqrt{5}}{6}, 0\right)$ and length of the latus rectum is $\frac{4}{9}$.

3 B. Question

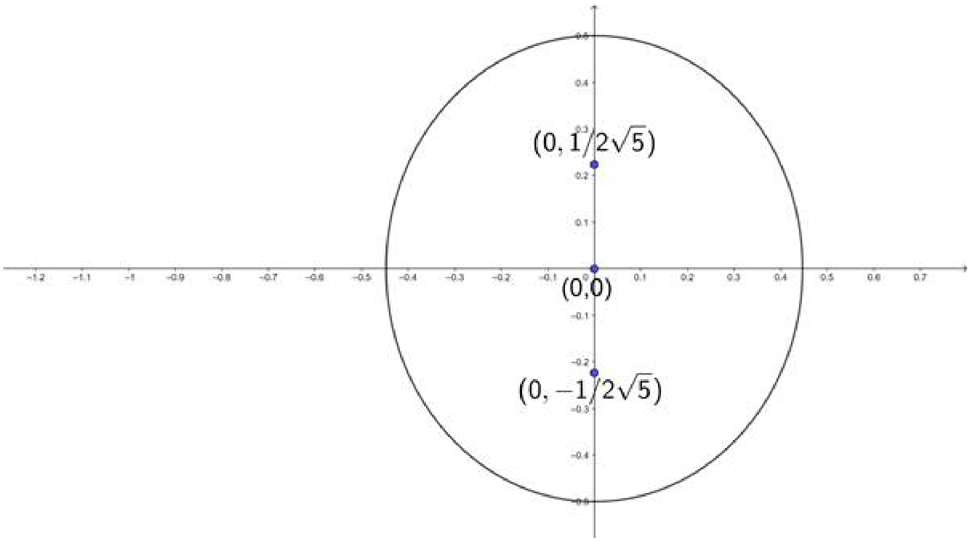
Find the eccentricity, coordinates of foci, length of the latus - rectum of the following ellipse:

$5x^2 + 4y^2 = 1$

Answer

Given the equation of the ellipse is $5x^2 + 4y^2 = 1$.

We need to find the eccentricity, coordinates of foci and length of latus rectum.



Given equation can be rewritten as $\frac{x^2}{\frac{1}{5}} + \frac{y^2}{\frac{1}{4}} = 1$.

We know for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(b^2>a^2)$

$\Rightarrow e = \sqrt{\frac{b^2-a^2}{b^2}}$

⇒ Coordinates of foci $(0,\pm be)$

⇒ Length of latus rectum $= \frac{2a^2}{b}$

Here $a^2 = \frac{1}{5}$ and $b^2 = \frac{1}{4}$, $b^2>a^2$

$\Rightarrow e = \sqrt{\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}}}$

$\Rightarrow e = \sqrt{\frac{\frac{1}{20}}{\frac{1}{4}}}$

$\Rightarrow e = \sqrt{\frac{1}{5}}$

$\Rightarrow \text{foci} = \left(0, \pm\frac{1}{2} \times \sqrt{\frac{1}{5}}\right)$

$\Rightarrow \text{foci} = \left(0, \pm\frac{1}{2\sqrt{5}}\right)$

⇒ Length of latus rectum (L) $= \frac{2\left(\frac{1}{5}\right)}{\frac{1}{2}}$

$\Rightarrow L = \frac{4}{5}$

∴ The eccentricity is $\sqrt{\frac{1}{5}}$, foci are $\left(0, \pm\frac{1}{2\sqrt{5}}\right)$ and length of the latus rectum is $\frac{4}{5}$.

3 C. Question

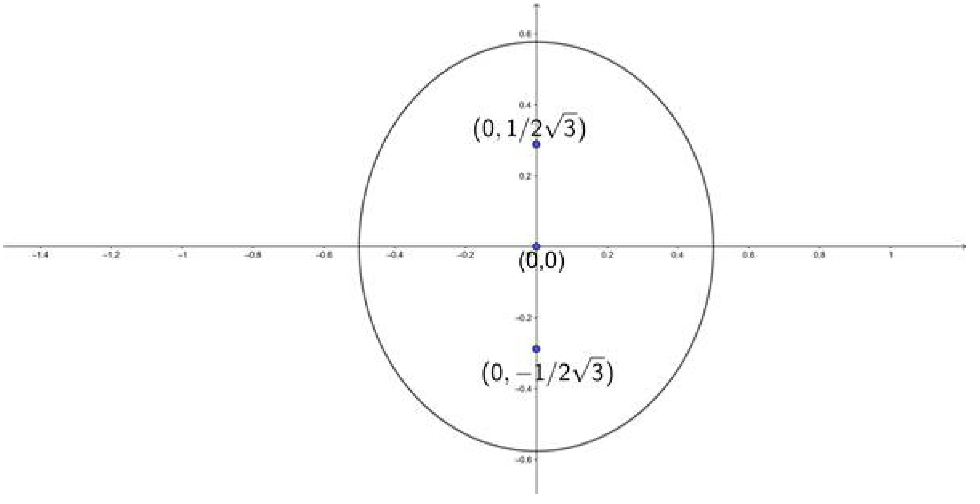
Find the eccentricity, coordinates of foci, length of the latus - rectum of the following ellipse:

$4x^2 + 3y^2 = 1$

Answer

Given the equation of the ellipse is $4x^2 + 3y^2 = 1$.

We need to find the eccentricity, coordinates of foci and length of latus rectum.



Given equation can be rewritten as $\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{3}} = 1$.

We know for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(b^2>a^2)$

$\Rightarrow e = \sqrt{\frac{b^2-a^2}{b^2}}$

⇒ Coordinates of foci $(0,\pm be)$

⇒ Length of latus rectum $= \frac{2a^2}{b}$

Here $a^2 = \frac{1}{4}$ and $b^2 = \frac{1}{3}$, $b^2>a^2$

$\Rightarrow e = \sqrt{\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}}}$

$\Rightarrow e = \sqrt{\frac{\frac{1}{12}}{\frac{1}{3}}}$

$\Rightarrow e = \sqrt{\frac{1}{4}}$

$\Rightarrow e = \frac{1}{2}$

$\Rightarrow \text{foci} = \left(0, \pm\frac{1}{\sqrt{3}} \times \frac{1}{2}\right)$

$\Rightarrow \text{foci} = \left(0, \pm\frac{1}{2\sqrt{3}}\right)$

⇒ Length of latus rectum (L) $= \frac{2\left(\frac{1}{4}\right)}{\frac{1}{\sqrt{3}}}$

$\Rightarrow L = \frac{\sqrt{3}}{2}$

∴ The eccentricity is $\frac{\sqrt{3}}{2}$, foci are $\left(0, \pm\frac{1}{2\sqrt{3}}\right)$ and length of the latus rectum is $\frac{\sqrt{3}}{2}$.

3 D. Question

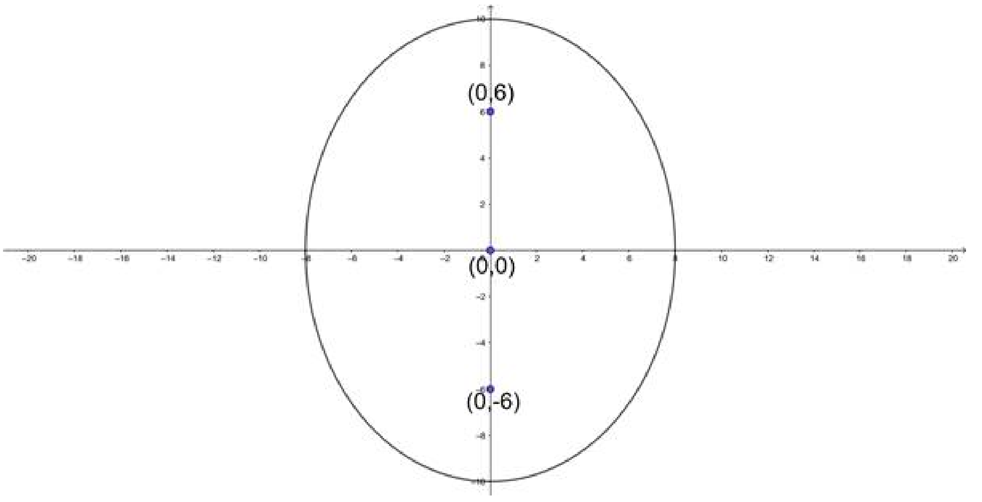
Find the eccentricity, coordinates of foci, length of the latus - rectum of the following ellipse:

$25x^2 + 16y^2 = 1600$

Answer

Given the equation of the ellipse is $25x^2 + 16y^2 = 1600$.

We need to find the eccentricity, coordinates of foci and length of latus rectum.



Given equation can be rewritten as $\frac{25x^2}{1600} + \frac{16y^2}{1600} = 1$

$\Rightarrow \frac{x^2}{64} + \frac{y^2}{100} = 1.$

We know for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (b^2 > a^2)$

$\Rightarrow e = \sqrt{\frac{b^2 - a^2}{b^2}}$

$\Rightarrow$ Coordinates of foci $(0, \pm be)$

$\Rightarrow$ Length of latus rectum $= \frac{2a^2}{b}$

Here $a^2 = 64$ and $b^2 = 100$, $b^2 > a^2$

$\Rightarrow e = \sqrt{\frac{100-64}{100}}$

$\Rightarrow e = \sqrt{\frac{36}{100}}$

$\Rightarrow e = \frac{6}{10}$

$\Rightarrow e = \frac{3}{5}$

$\Rightarrow \text{foci} = \left(0, \pm 10 \times \frac{3}{5}\right)$

$\Rightarrow$ foci $= (0, \pm 6)$

$\Rightarrow$ Length of latus rectum (L) $= \frac{2(64)}{100}$

$\Rightarrow L = \frac{32}{25}$

∴ The eccentricity is $\frac{3}{5}$, foci are (0,±6) and length of the latus rectum is $\frac{32}{25}$.

3 E. Question

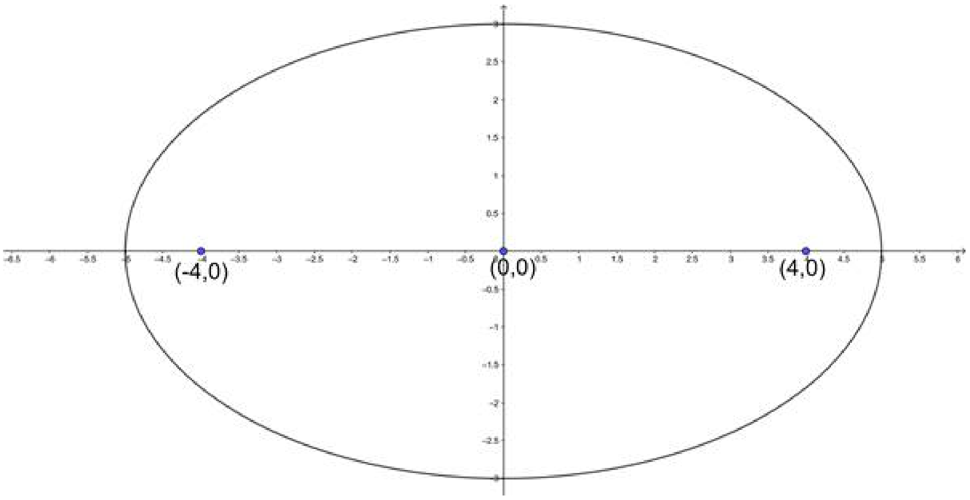
Find the eccentricity, coordinates of foci, length of the latus - rectum of the following ellipse:

$9x^2 + 25y^2 = 225$

Answer

Given the equation of the ellipse is $9x^2 + 25y^2 = 225$.

We need to find the eccentricity, coordinates of foci and length of latus rectum.



Given equation can be rewritten as $\frac{9x^2}{225} + \frac{25y^2}{225} = 1$

$\Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1.$

We know for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a^2>b^2)$

$\Rightarrow e = \sqrt{\frac{a^2-b^2}{a^2}}$

⇒ Coordinates of foci $(\pm ae,0)$

⇒ Length of latus rectum $= \frac{2b^2}{a}$

Here $a^2 = 25$ and $b^2 = 9$, $a^2>b^2$

$\Rightarrow e = \sqrt{\frac{25-9}{25}}$

$\Rightarrow e = \sqrt{\frac{16}{25}}$

$\Rightarrow e = \frac{4}{5}$

$\Rightarrow \text{foci} = \left(\pm 5 \times \frac{4}{5}, 0\right)$

$\Rightarrow \text{foci} = (\pm 4, 0)$

⇒ Length of latus rectum (L) $= \frac{2(9)}{5}$

$\Rightarrow L = \frac{18}{5}$

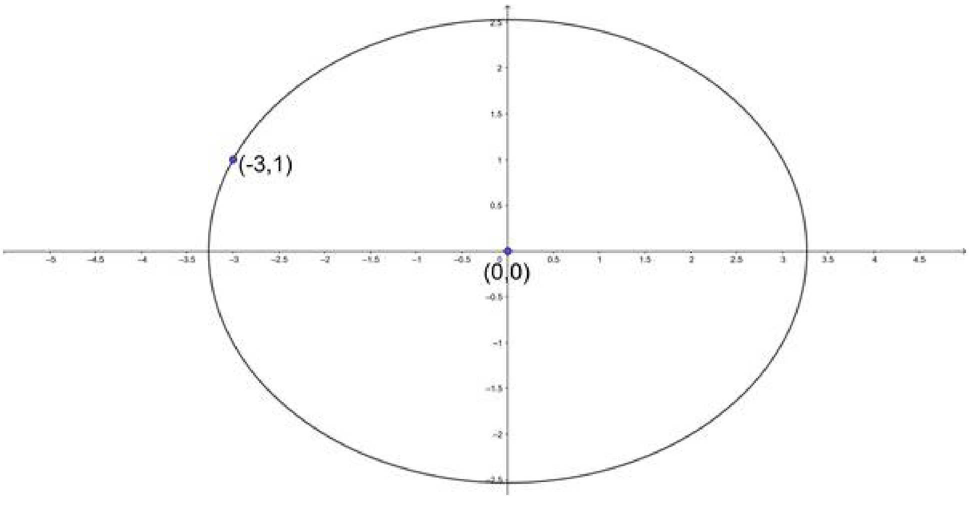
∴ The eccentricity is $\frac{4}{5}$, foci are $(\pm 4,0)$ and length of the latus rectum is $\frac{18}{5}$.

4. Question

Find the equation to the ellipse (referred to its axes as the axes of x and y respectively) which passes through the point (- 3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$.

Answer

Given that we need to find the equation of the ellipse (whose axes are x = 0 and y = 0) which passes through the point (- 3,1) and has eccentricity $\sqrt{\frac{2}{5}}$.



We know that the equation of the ellipse whose axes are x and y - axis is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. ..... - - - - - (1)

Let us assume $a^2 > b^2$.

We know that eccentricity(e) = $\sqrt{\frac{a^2 - b^2}{a^2}}$

$\Rightarrow \sqrt{\frac{2}{5}} = \sqrt{\frac{a^2 - b^2}{a^2}}$

$\Rightarrow \frac{2}{5} = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{3}{5}$

$\Rightarrow b^2 = \frac{3a^2}{5}$ .... (2)

Substituting (2) in (1) we get,

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{\frac{3a^2}{5}} = 1$

$\Rightarrow \frac{x^2}{a^2} + \frac{5y^2}{3a^2} = 1$

$\Rightarrow 3x^2 + 5y^2 = 3a^2$

This curve passes through the point (- 3,1). Substituting in the curve we get,

$\Rightarrow 3(-3)^2 + 5(1)^2 = 3a^2$

$\Rightarrow 3(9) + 5 = 3a^2$

$\Rightarrow 32 = 3a^2$

$\Rightarrow a^2 = \frac{32}{3}$

$\Rightarrow b^2 = \frac{3\left(\frac{32}{3}\right)}{5}$

$\Rightarrow b^2 = \frac{32}{5}$

The equation of the ellipse is:

$\Rightarrow \frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$

$\Rightarrow \frac{3x^2}{32} + \frac{5y^2}{32} = 1$

$\Rightarrow 3x^2 + 5y^2 = 32$

∴ The equation of the ellipse is $3x^2 + 5y^2 = 32$.

5 A. Question

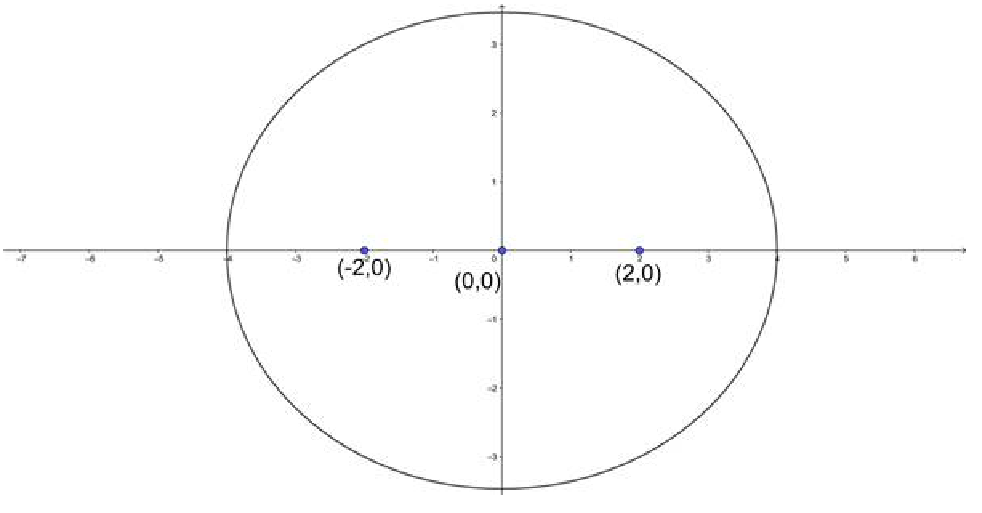
find the equation of the ellipse in the following cases:

eccentricity $e = \frac{1}{2}$ and foci (± 2, 0)

Answer

Given that we need to find the equation of the ellipse whose eccentricity is $\frac{1}{2}$ and foci (±2,0).

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2 > b^2$).



We know that eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{1}{2} = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{1}{4} = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{3}{4}$

$\Rightarrow b^2 = \frac{3a^2}{4}$

We know that foci = (±ae,0)

$\Rightarrow ae = 2$

$\Rightarrow a\left(\frac{1}{2}\right) = 2$

$\Rightarrow a = 4$

$\Rightarrow a^2 = 16$

$\Rightarrow b^2 = \frac{3(16)}{4}$

$\Rightarrow b^2 = 12$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$

$\Rightarrow \frac{3x^2 + 4y^2}{48} = 1$

$\Rightarrow 3x^2 + 4y^2 = 48$

$\therefore$ The equation of the ellipse is $3x^2 + 4y^2 = 48$.

5 B. Question

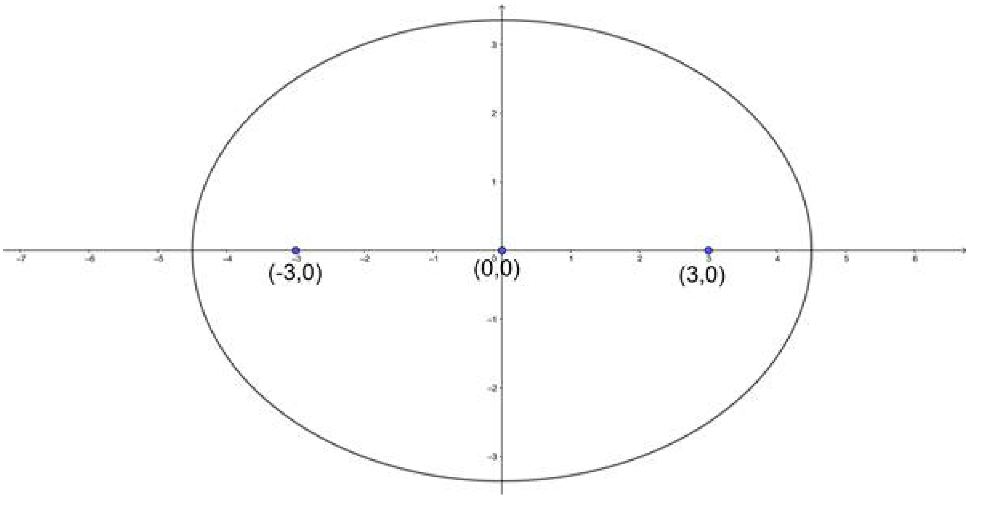
find the equation of the ellipse in the following cases:

eccentricity $e = \frac{2}{3}$ and length of latus - rectum = 5

Answer

Given that we need to find the equation of the ellipse whose eccentricity is $\frac{2}{3}$ and length of latus rectum is 5.

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2 > b^2$).



We know that eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{2}{3} = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{4}{9} = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{5}{9}$

$\Rightarrow b^2 = \frac{5a^2}{9}$

We know that the length of the latus rectum is $\frac{2b^2}{a}$.

$\Rightarrow \frac{2b^2}{a} = 5$

$\Rightarrow b^2 = \frac{5a}{2}$

$\Rightarrow \frac{5a^2}{9} = \frac{5a}{2}$

$\Rightarrow \frac{a}{9} = \frac{1}{2}$

$\Rightarrow a = \frac{9}{2}$

$\Rightarrow a^2 = \frac{81}{4}$

$\Rightarrow b^2 = \frac{5\left(\frac{81}{4}\right)}{9}$

$\Rightarrow b^2 = \frac{45}{4}$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$

$\Rightarrow \frac{4x^2}{81} + \frac{4y^2}{45} = 1$

$\Rightarrow \frac{(20x^2 + 36y^2)}{405} = 1$

$\Rightarrow 20x^2 + 36y^2 = 405$

$\therefore$ The equation of the ellipse is $20x^2 + 36y^2 = 405$.

5 C. Question

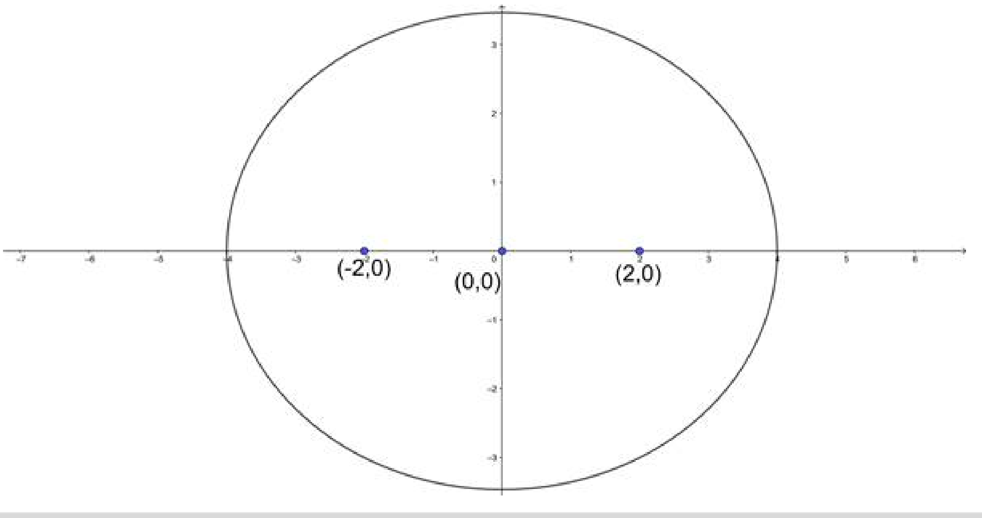
find the equation of the ellipse in the following cases:

eccentricity $e = \frac{1}{2}$ and semi - major axis = 4

Answer

Given that we need to find the equation of the ellipse whose eccentricity is $\frac{1}{2}$ and the semi - major axis is 4.

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2 > b^2$).



We know that eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{1}{2} = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{1}{4} = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{3}{4}$

$\Rightarrow b^2 = \frac{3a^2}{4}$

We know that the length of the semi - major axis is a

$\Rightarrow$ a = 4

$\Rightarrow a^2 = 16$

$\Rightarrow b^2 = \frac{3(16)}{4}$

$\Rightarrow b^2 = 12$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$

$\Rightarrow \frac{3x^2 + 4y^2}{48} = 1$

$\Rightarrow 3x^2 + 4y^2 = 48$

∴ The equation of the ellipse is $3x^2 + 4y^2 = 48$.

5 D. Question

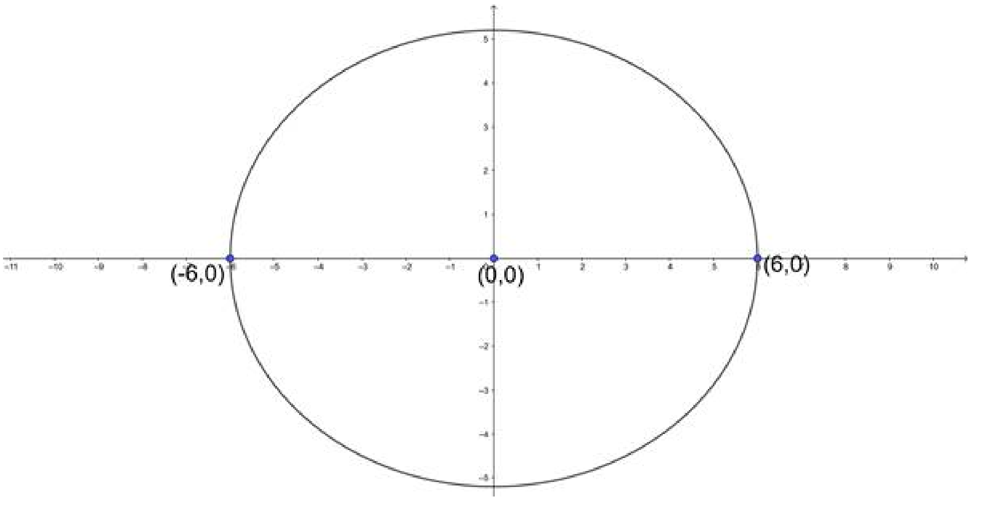
find the equation of the ellipse in the following cases:

eccentricity $e = \frac{1}{2}$ and major axis = 12

Answer

Given that we need to find the equation of the ellipse whose eccentricity is $\frac{1}{2}$ and the major axis is 12.

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2>b^2$).



We know that eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{1}{2} = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{1}{4} = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{3}{4}$

$\Rightarrow b^2 = \frac{3a^2}{4}$

We know that length of ,ajor axis is 2a.

⇒ 2a = 12

⇒ a = 6

⇒ a^2 = 36

$\Rightarrow b^2 = \frac{3(36)}{4}$

⇒ b^2 = 27

The equation of the ellipse is

$\Rightarrow \frac{x^2}{36} + \frac{y^2}{27} = 1$

$\Rightarrow \frac{3x^2 + 4y^2}{108} = 1$

⇒ $3x^2 + 4y^2 = 108$

∴ The equation of the ellipse is $3x^2 + 4y^2 = 108$.

5 E. Question

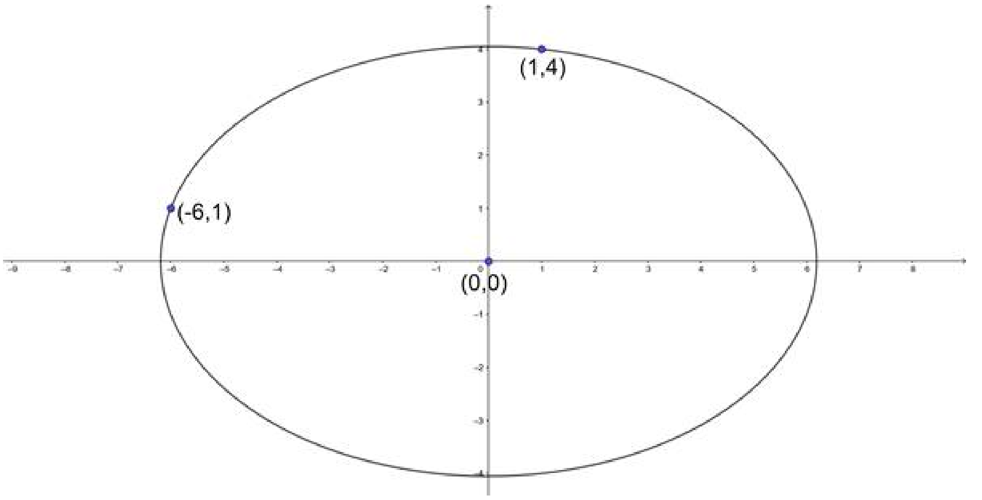
find the equation of the ellipse in the following cases:

The ellipse passes through (1, 4) and (- 6, 1)

Answer

Given that we need to find the equation of the ellipse passing through the points (1,4) and (- 6,1).

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2>b^2$). .... (1)



Substituting the point (1,4) in (1) we get

$\Rightarrow \frac{1^2}{a^2} + \frac{4^2}{b^2} = 1$

$\Rightarrow \frac{1}{a^2} + \frac{16}{b^2} = 1$

$\Rightarrow \frac{b^2 + 16a^2}{a^2 b^2} = 1$

$\Rightarrow b^2 + 16a^2 = a^2\, b^2$ ..... - - (2)

Substituting the point (- 6,1) in (1) we get

$\Rightarrow \frac{(-6)^2}{a^2} + \frac{1^2}{b^2} = 1$

$\Rightarrow \frac{36}{a^2} + \frac{1}{b^2} = 1$

$\Rightarrow \frac{36b^2 + a^2}{a^2 b^2} = 1$

$\Rightarrow a^2 + 36b^2 = a^2b^2$ ..... - - (3)

(3)×16 - (2)

$\Rightarrow (16a^2 + 576b^2) - (b^2 + 16a^2) = (16a^2b^2 - a^2b^2)$

$\Rightarrow 575b^2 = 15a^2b^2$

$\Rightarrow 15a^2 = 575$

$\Rightarrow a^2 = \frac{115}{3}$

From (2)

$\Rightarrow b^2 + 16\left(\frac{115}{3}\right) = b^2\left(\frac{115}{3}\right)$

$\Rightarrow b^2\left(\frac{112}{3}\right) = \frac{1840}{3}$

$\Rightarrow b^2 = \frac{115}{7}$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{\frac{115}{3}} + \frac{y^2}{\frac{115}{7}} = 1$

$\Rightarrow \frac{3x^2}{115} + \frac{7y^2}{115} = 1$

$\Rightarrow 3x^2 + 7y^2 = 115$

∴ The equation of the ellipse is $3x^2 + 7y^2 = 115$.

5 F. Question

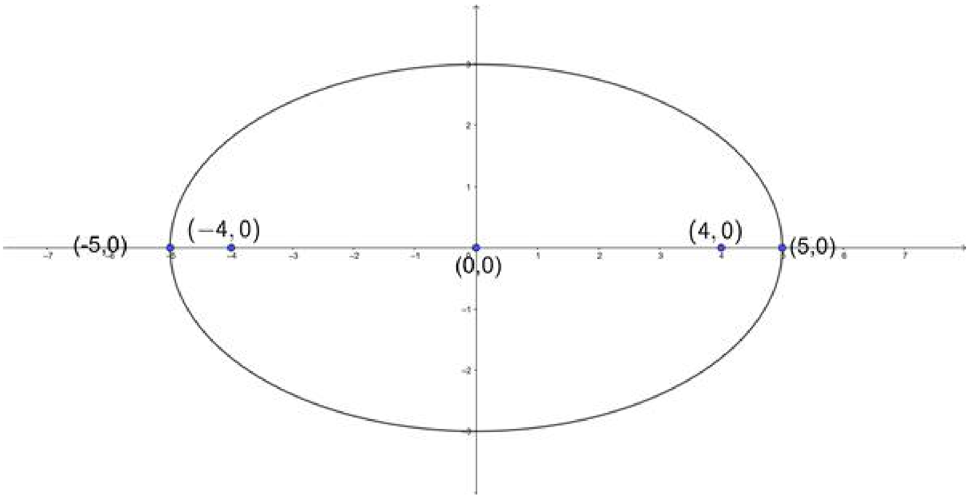
find the equation of the ellipse in the following cases:

Vertices (± 5, 0), foci (± 4, 0)

Answer

Given that we need to find the equation of the ellipse whose vertices are (±5,0) and foci (±4,0).

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2>b^2$).



We know that vertices of the ellipse are (±a,0)

⇒ a = 5

⇒ $a^2 = 25$

We know that foci = (±ae,0)

⇒ ae = 4

⇒ 5e = 4

⇒ $e = \frac{4}{5}$

We know that eccentricity $e = \sqrt{\frac{a^2-b^2}{a^2}}$

⇒ $\frac{4}{5} = \sqrt{\frac{25-b^2}{25}}$

⇒ $\frac{16}{25} = 1 - \frac{b^2}{25}$

⇒ $\frac{b^2}{25} = \frac{9}{25}$

⇒ $b^2 = 9$

The equation of the ellipse is

⇒ $\frac{x^2}{25} + \frac{y^2}{9} = 1$

⇒ $\frac{9x^2 + 25y^2}{225} = 1$

⇒ $9x^2 + 25y^2 = 225$

∴ The equation of the ellipse is $9x^2 + 25y^2 = 225$.

5 G. Question

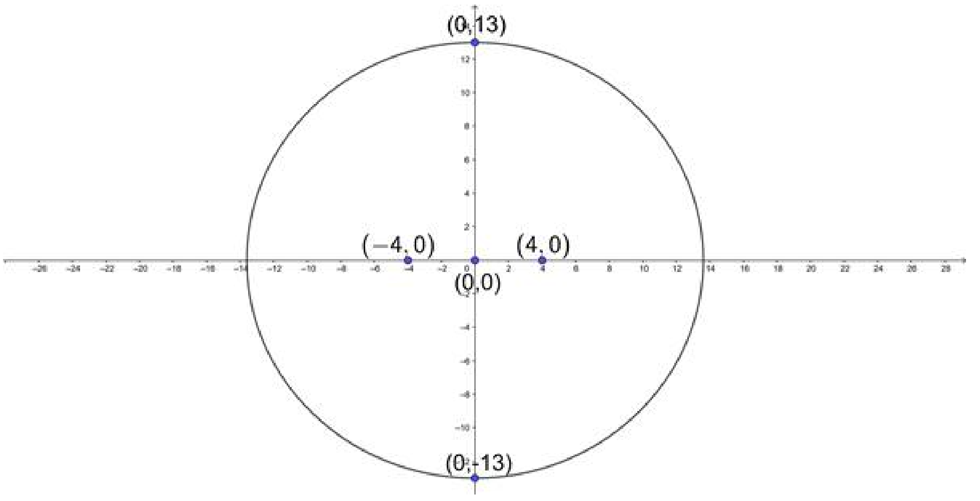
find the equation of the ellipse in the following cases:

Vertices (0, ±13), foci (±4, 0)

Answer

Given that we need to find the equation of the ellipse whose vertices are (0,±13) and foci (±4,0).

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a^2>b^2)$.



We know that vertices of the ellipse are (0,±b)

⇒ b = 13

⇒ b^2 = 169

We know that foci = (±ae,0)

⇒ ae = 4

We know that eccentricity $e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow 4 = a\sqrt{\frac{a^2-169}{a^2}}$

⇒ 16 = a^2 - 169

⇒ a^2 = 185

The equation of the ellipse is

$\Rightarrow \frac{x^2}{185} + \frac{y^2}{169} = 1$

∴ The equation of the ellipse is $\frac{x^2}{185} + \frac{y^2}{165} = 1$.

5 H. Question

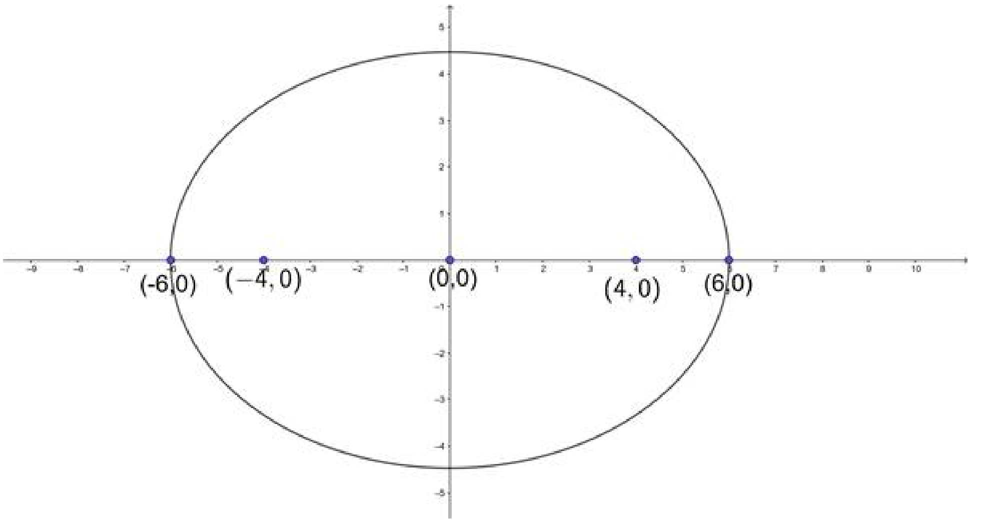
find the equation of the ellipse in the following cases:

Vertices (± 6, 0), foci (± 4, 0)

Answer

Given that we need to find the equation of the ellipse whose vertices are (±6,0) and foci (±4,0).

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a^2>b^2).



We know that vertices of the ellipse are (±a,0)

⇒ $a = 6$

⇒ $a^2 = 36$

We know that foci = (±ae,0)

⇒ $ae = 4$

⇒ $6e = 4$

⇒ $e = \frac{2}{3}$

We know that eccentricity $e = \sqrt{\frac{a^2-b^2}{a^2}}$

⇒ $\frac{2}{3} = \sqrt{\frac{36-b^2}{36}}$

⇒ $\frac{4}{9} = 1 - \frac{b^2}{36}$

⇒ $\frac{b^2}{36} = \frac{5}{9}$

⇒ $b^2 = 20$

The equation of the ellipse is

⇒ $\frac{x^2}{36} + \frac{y^2}{20} = 1$

⇒ $\frac{5x^2 + 9y^2}{180} = 1$

⇒ $5x^2 + 9y^2 = 180$

∴ The equation of the ellipse is $5x^2 + 9y^2 = 180$.

5 I. Question

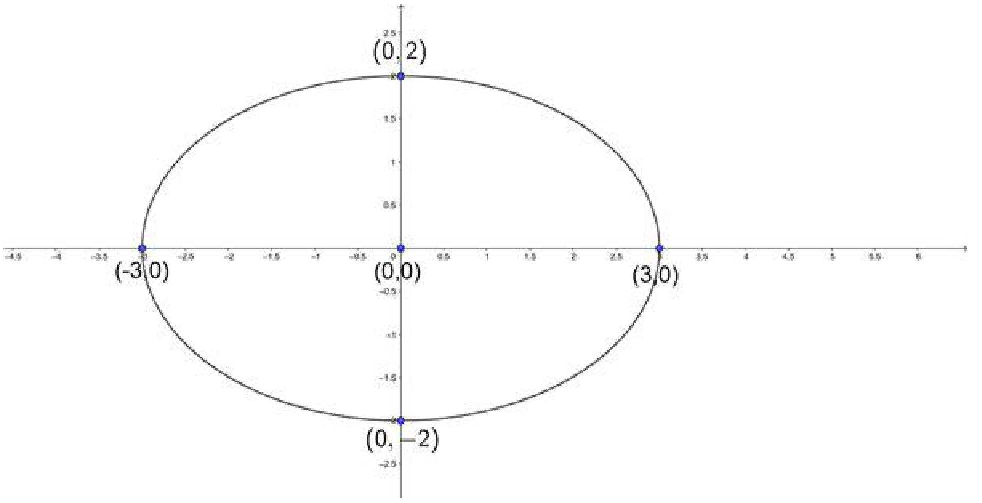
find the equation of the ellipse in the following cases:

Ends of the major axis (± 3, 0), and of the minor axis (0, ±2)

Answer

Given that we need to find the equation of the ellipse whose ends of the major axis is (±3,0) and ends of the minor axis is (0,±2).

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2 > b^2$).



We know that ends of the major axis of the ellipse are (±a,0)

⇒ $a = 3$

⇒ $a^2 = 9$

We know that ends of minor axis of the ellipse are (0,±b)

⇒ $b = 2$

⇒ $b^2 = 4$

The equation of the ellipse is

⇒ $\frac{x^2}{9} + \frac{y^2}{4} = 1$

⇒ $\frac{4x^2 + 9y^2}{36} = 1$

⇒ $4x^2 + 9y^2 = 36$

∴ The equation of the ellipse is $4x^2 + 9y^2 = 36$.

5 J. Question

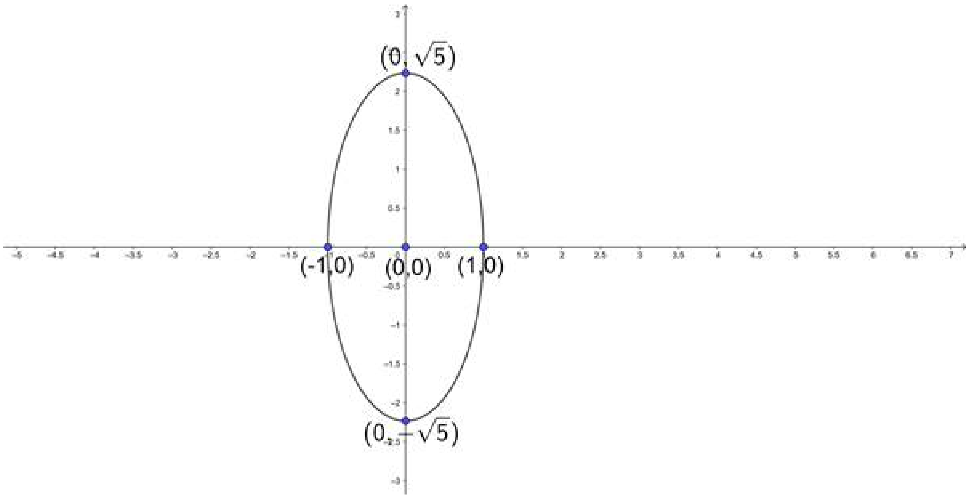
find the equation of the ellipse in the following cases:

Ends of the major axis $(0, \pm\sqrt{5})$, ends of the minor axis (±1,0)

Answer

Given that we need to find the equation of the ellipse whose ends of major axis are $(0, \pm\sqrt{5})$ and ends of the minor axis are (±1,0).

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($b^2 > a^2$).



We know that ends of the major axis of the ellipse are (0,±b)

$\Rightarrow b = \sqrt{5}$

$\Rightarrow b^2 = 5$

We know that ends of the minor axis of the ellipse are (±a,0)

$\Rightarrow a = 1$

$\Rightarrow a^2 = 1$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{1} + \frac{y^2}{5} = 1$

$\Rightarrow \frac{5x^2 + y^2}{5} = 1$

$\Rightarrow 5x^2 + y^2 = 5$

∴ The equation of the ellipse is $5x^2 + y^2 = 5$.

5 K. Question

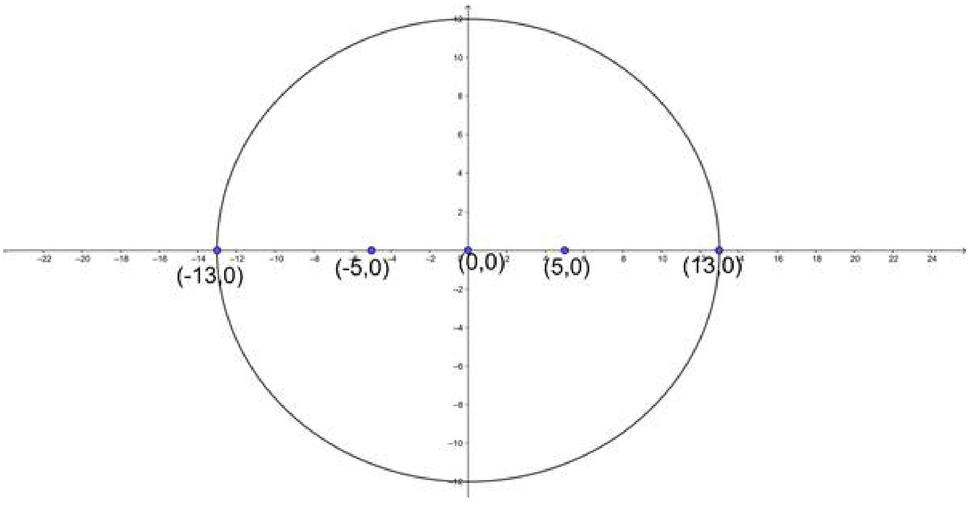
find the equation of the ellipse in the following cases:

Length of major axis 26, foci (±5, 0)

Answer

Given that we need to find the equation of the ellipse whose length of major axis is 26 and foci (±5,0).

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2>b^2$).



We know that length of the major axis is 2a

⇒ 2a = 26

⇒ a = 13

⇒ a^2 = 169

We know that foci = (±ae,0)

⇒ ae = 5

⇒ 13e = 5

⇒ $e = \frac{5}{13}$

We know that eccentricity $e = \sqrt{\frac{a^2-b^2}{a^2}}$

⇒ $\frac{5}{13} = \sqrt{\frac{169-b^2}{169}}$

⇒ $\frac{25}{169} = 1 - \frac{b^2}{169}$

⇒ $\frac{b^2}{169} = \frac{144}{169}$

⇒ b^2 = 144

The equation of the ellipse is

⇒ $\frac{x^2}{169} + \frac{y^2}{144} = 1$

∴ The equation of the ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$.

5 L. Question

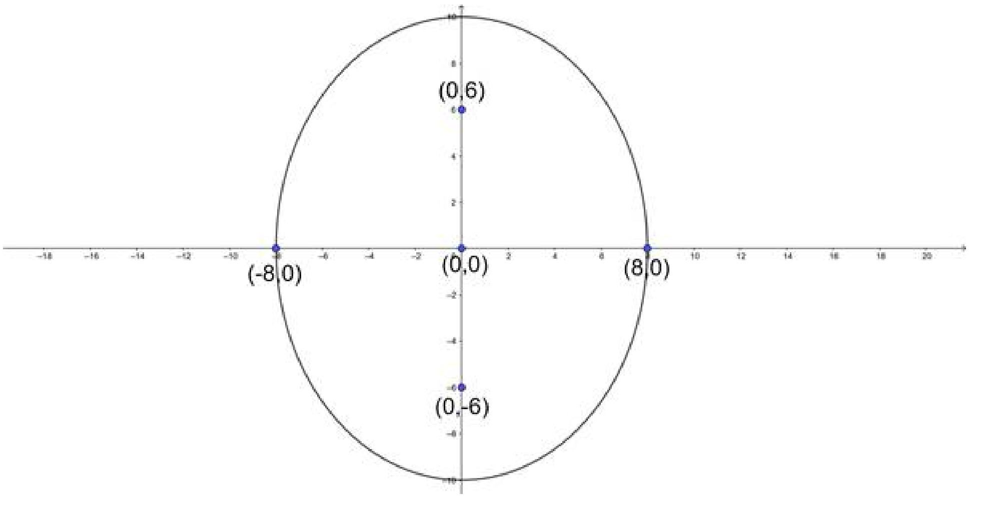
find the equation of the ellipse in the following cases:

Length of minor axis 16 foci (0, ± 6)

Answer

Given that we need to find the equation of the ellipse whose length of the minor axis is 16 and foci (0,±6).

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (b^2>a^2).



We know that the length of the minor axis of the ellipse is 2a,

⇒ 2a = 16

⇒ a = 8

⇒ a^2 = 64

We know that foci = (0,±be)

⇒ be = 6

We know that eccentricity $e = \sqrt{\frac{b^2 - a^2}{b^2}}$

$\Rightarrow 6 = b\sqrt{\frac{b^2 - 64}{b^2}}$

⇒ 36 = b^2 - 64

⇒ b^2 = 100

The equation of the ellipse is

$\Rightarrow \frac{x^2}{64} + \frac{y^2}{100} = 1$

∴ The equation of the ellipse is $\frac{x^2}{64} + \frac{y^2}{100} = 1$.

5 M. Question

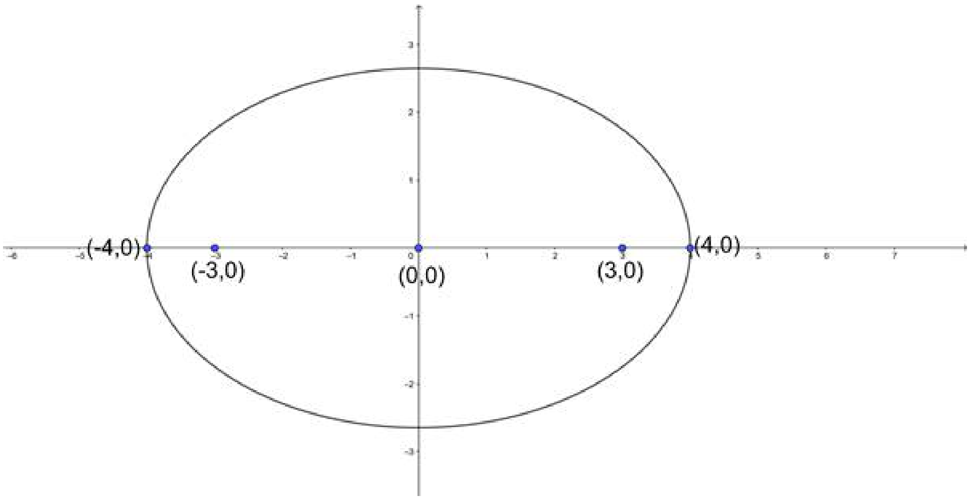
find the equation of the ellipse in the following cases:

Foci (± 3, 0), a = 4

Answer

Given that we need to find the equation of the ellipse whose foci are (±3,0) and a = 4.

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2 > b^2$).



⇒ a = 4

⇒ a^2 = 16

We know that foci = (±ae,0)

⇒ ae = 3

⇒ 4e = 3

$\Rightarrow e = \frac{3}{4}$

We know that eccentricity $e = \sqrt{\frac{a^2 - b^2}{a^2}}$

$\Rightarrow \frac{3}{4} = \sqrt{\frac{16-b^2}{16}}$

$\Rightarrow \frac{9}{16} = 1 - \frac{b^2}{16}$

$\Rightarrow \frac{b^2}{16} = \frac{7}{16}$

$\Rightarrow b^2 = 7$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{7} = 1$

$\Rightarrow \frac{7x^2 + 16y^2}{112} = 1$

$\Rightarrow 7x^2 + 16y^2 = 112$

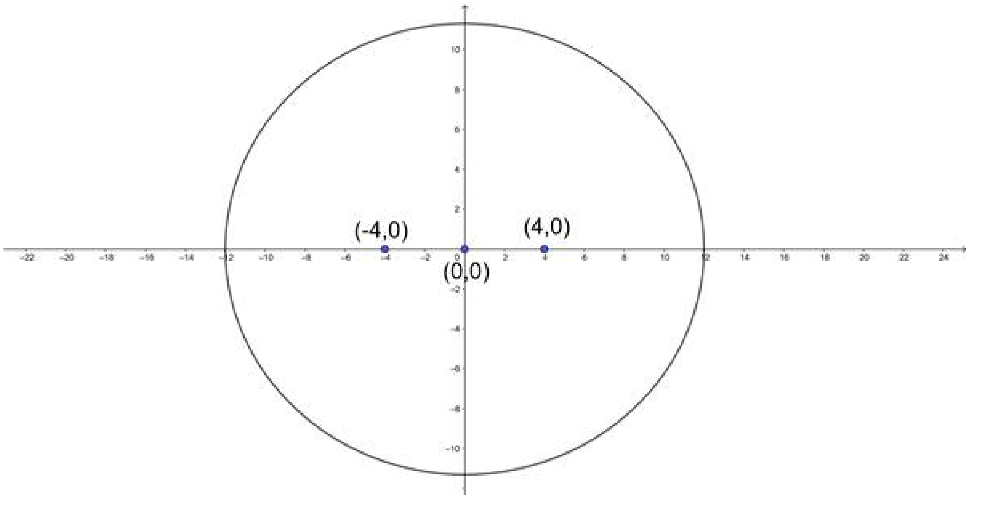
∴ The equation of the ellipse is $7x^2 + 16y^2 = 112$.

6. Question

Find the equation of the ellipse whose foci are (4, 0) and (- 4, 0), eccentricity = 1/3.

Answer

Given that we need to find the equation of the ellipse whose eccentricity is $\frac{1}{3}$ and foci (±4,0).



Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2>b^2$).

We know that eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{1}{3} = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{1}{9} = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{8}{9}$

$\Rightarrow b^2 = \frac{8a^2}{9}$

We know that foci = (±ae,0)

$\Rightarrow ae = 4$

$\Rightarrow a\left(\frac{1}{3}\right) = 4$

$\Rightarrow a = 12$

$\Rightarrow a^2 = 144$

$\Rightarrow b^2 = \frac{8(144)}{9}$

$\Rightarrow b^2 = 128$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{144} + \frac{y^2}{128} = 1$

$\therefore$ The equation of the ellipse is $\frac{x^2}{144} + \frac{y^2}{128} = 1$.

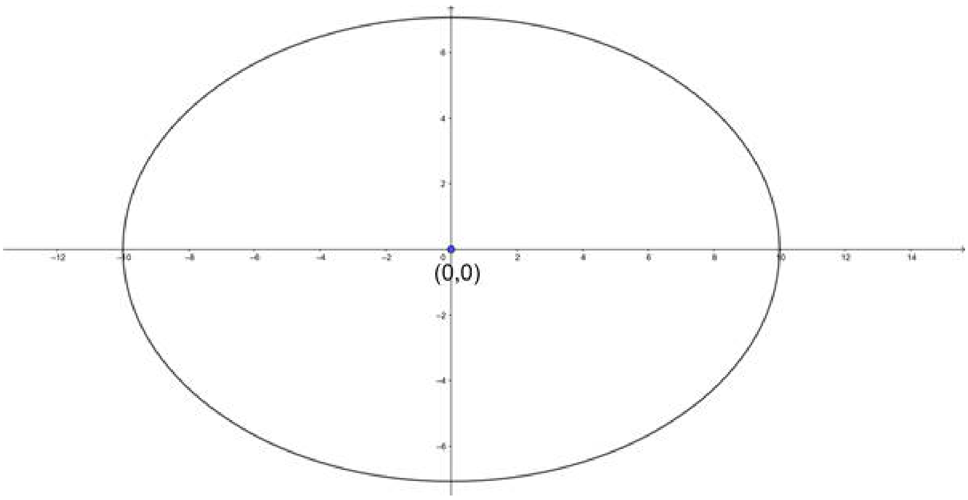
7. Question

Find the equation of the ellipse in the standard form whose minor axis is equal to the distance between foci and whose latus - rectum is 10.

Answer

Given that we need to find the equation of the ellipse whose minor axis is equal to the distance between foci and length of latus rectum is 10.

Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2 > b^2$).



We know that length of the minor axis is 2b and distance between the foci is 2ae.

We know that eccentricity $e = \sqrt{\frac{a^2 - b^2}{a^2}}$

$\Rightarrow 2b = 2ae$

$\Rightarrow b = ae$

$\Rightarrow b = a\sqrt{\frac{a^2 - b^2}{a^2}}$

$\Rightarrow b^2 = a^2 - b^2$

$\Rightarrow a^2 = 2b^2$ ..... - - - - (1)

We know that the length of the latus rectum is $\frac{2b^2}{a}$.

$\Rightarrow \frac{2b^2}{a} = 10$

From (1)

$\Rightarrow \frac{a^2}{a} = 10$

⇒ a = 10

⇒ $a^2 = 100$

$\Rightarrow b^2 = \frac{100}{2}$

⇒ $b^2 = 50$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{100} + \frac{y^2}{50} = 1$

$\Rightarrow \frac{x^2 + 2y^2}{100} = 1$

⇒ $x^2 + 2y^2 = 100$

∴ The equation of the ellipse is $x^2 + 2y^2 = 100$.

8. Question

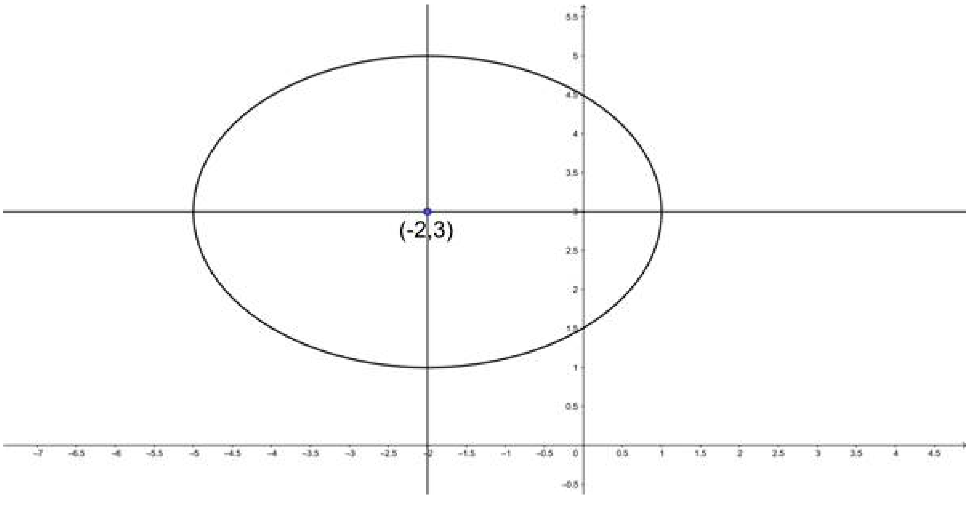
Find the equation of the ellipse whose centre is (- 2, 3) and whose semi - axis are 3 and 2 when the major axis is (i) parallel to x - axis (ii) parallel to the y - axis.

Answer

Given that we need to find the equation of the ellipse whose centre is (- 2,3) and whose semi - axis are 3 and 2.

(i) If major axis is parallel to the x - axis.

We know that the equation of the ellipse with centre (p,q) is given by $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$.



Since major axis is parallel to x - axis $a^2 > b^2$.

So, a = 3 and b = 2.

⇒ $a^2 = 9$

⇒ $b^2 = 4$

The equation of the ellipse is

$\Rightarrow \frac{(x+2)^2}{9} + \frac{(y-3)^2}{4} = 1$

$\Rightarrow \frac{4(x+2)^2+9(y-3)^2}{36} = 1$

$\Rightarrow 4(x^2 + 4x + 4) + 9(y^2 - 6y + 9) = 36$

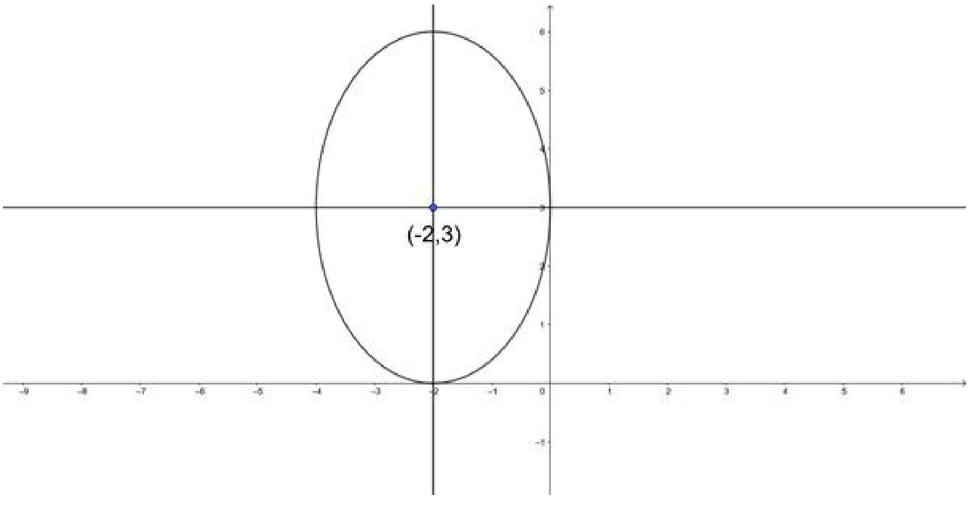
$\Rightarrow 4x^2 + 16x + 16 + 9y^2 - 54y + 81 = 36$

$\Rightarrow 4x^2 + 9y^2 + 16x - 54y + 61 = 0$

∴ The equation of the ellipse is $4x^2 + 9y^2 + 16x - 54y + 61 = 0$.

(ii) If major axis is parallel to the y - axis.

We know that the equation of the ellipse with centre (p,q) is given by $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$.



Since major axis is parallel to y - axis $b^2>a^2$.

So, a = 2 and b = 3.

$\Rightarrow a^2 = 4$

$\Rightarrow b^2 = 9$

The equation of the ellipse is

$\Rightarrow \frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$

$\Rightarrow \frac{9(x+2)^2+4(y-3)^2}{36} = 1$

$\Rightarrow 9(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = 36$

$\Rightarrow 9x^2 + 36x + 36 + 4y^2 - 24y + 36 = 36$

$\Rightarrow 9x^2 + 4y^2 + 36x - 24y + 36 = 0$

∴ The equation of the ellipse is $9x^2 + 4y^2 + 36x - 24y + 36 = 0$.

9. Question

Find the eccentricity of an ellipse whose latus - rectum is

(i) Half of its minor axis

(ii) Half of its major axis

Answer

Given that we need to find the eccentricity of an ellipse.

(i) If latus - rectum is half of its minor axis

We know that the length of the semi - minor axis is b and the length of the latus - rectum is $\frac{2b^2}{a}$.

$\Rightarrow \frac{2b^2}{a} = b$

⇒ a = 2b .... (1)

We know that eccentricity of an ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$

From (1)

$\Rightarrow e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{(2b)^2-b^2}{(2b)^2}}$

$\Rightarrow e = \sqrt{\frac{4b^2-b^2}{4b^2}}$

$\Rightarrow e = \sqrt{\frac{3}{4}}$

$\Rightarrow e = \frac{\sqrt{3}}{2}$.

(ii) If latus - rectum is half of its major axis

We know that the length of the semi - major axis is a and the length of the latus - rectum is $\frac{2b^2}{a}$.

$\Rightarrow \frac{2b^2}{a} = a$

⇒ $a^2 = 2b^2$ .... (1)

We know that eccentricity of an ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$

From (1)

$\Rightarrow e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{2b^2-b^2}{2b^2}}$

$\Rightarrow e = \sqrt{\frac{b^2}{2b^2}}$

$\Rightarrow e = \sqrt{\frac{1}{2}}$

$\Rightarrow e = \frac{1}{\sqrt{2}}$.

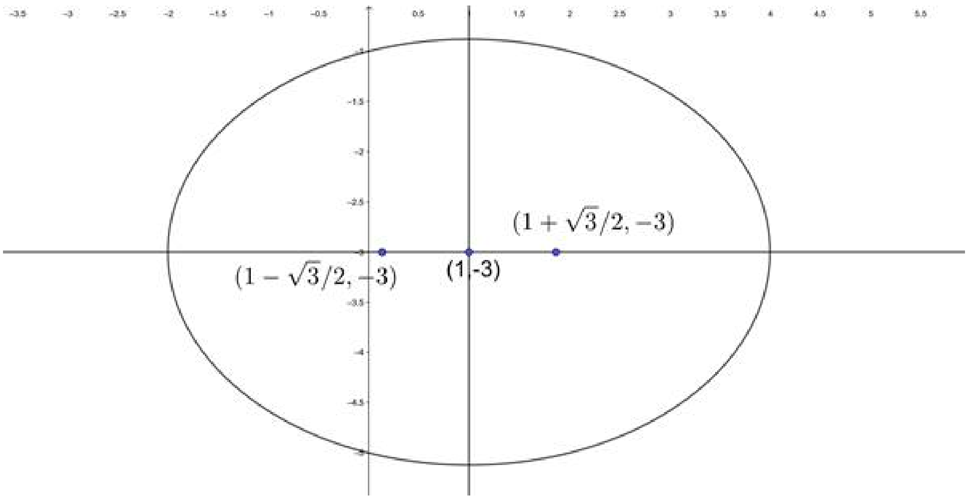
10 A. Question

Find the centre, the lengths of the axes, eccentricity, foci of the following ellipse:

$x^2 + 2y^2 - 2x + 12y + 10 = 0$

Answer

Given that we need to find the centre, lengths of axes, eccentricity and foci of the ellipse $x^2 + 2y^2 - 2x + 12y + 10 = 0$.



⇒ $x^2 + 2y^2 - 2x + 12y + 10 = 0$

⇒ $(x^2 - 2x + 1) + 2(y^2 + 6y + 9) - 9 = 0$

⇒ $(x - 1)^2 + 2(y + 3)^2 = 9$

⇒ $\frac{(x-1)^2}{9} + \frac{2(y+3)^2}{9} = 1$

⇒ $\frac{(x-1)^2}{9} + \frac{(y+3)^2}{\frac{9}{2}} = 1$

Comparing with the standard form $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

⇒ Centre = (p,q) = (1, - 3)

Here $a^2 > b^2$

⇒ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

⇒ $e = \sqrt{\frac{9-\frac{9}{2}}{9}}$

⇒ $e = \sqrt{\frac{\frac{9}{2}}{9}}$

⇒ $e = \sqrt{\frac{1}{2}}$

⇒ $e = \frac{1}{\sqrt{2}}$

Length of the major axis 2a = 2(3) = 6

Length of the minor axis 2b = $2\left(\frac{3}{\sqrt{2}}\right) = 3\sqrt{2}$

⇒ Foci = (p±ae,q)

⇒ Foci = $\left(\left(1 \pm \left(3 \times \frac{1}{\sqrt{2}}\right)\right), -3\right)$

⇒ Foci = $\left(\left(1 \pm \frac{3}{\sqrt{2}}\right), -3\right)$

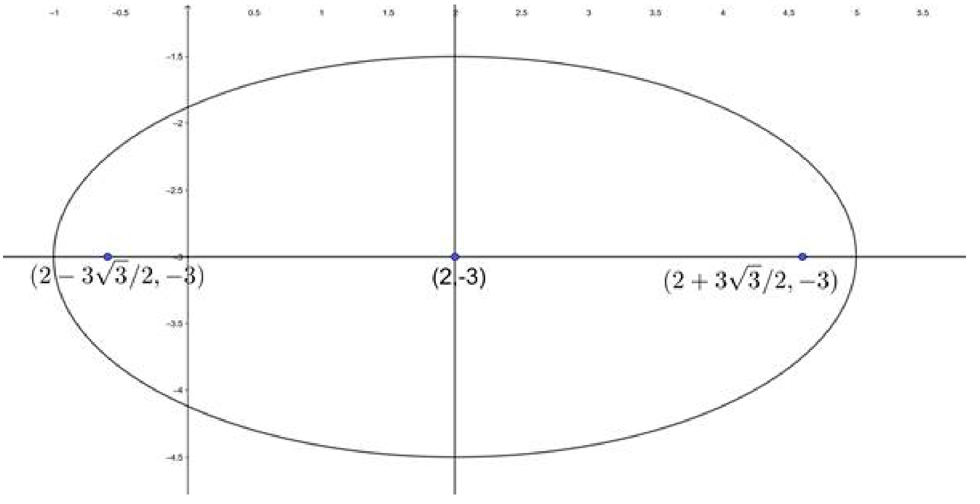
10 B. Question

Find the centre, the lengths of the axes, eccentricity, foci of the following ellipse:

$x^2 + 4y^2 - 4x + 24y + 31 = 0$

Answer

Given that we need to find the centre, lengths of axes, eccentricity and foci of the ellipse $x^2 + 4y^2 - 4x + 24y + 31 = 0$.



$\Rightarrow x^2 + 4y^2 - 4x + 24y + 31 = 0$

$\Rightarrow (x^2 - 4x + 4) + 4(y^2 + 6y + 9) - 9 = 0$

$\Rightarrow (x - 2)^2 + 4(y + 3)^2 = 9$

$\Rightarrow \frac{(x-2)^2}{9} + \frac{4(y+3)^2}{9} = 1$

$\Rightarrow \frac{(x-2)^2}{9} + \frac{(y+3)^2}{\frac{9}{4}} = 1$

Comparing with the standard form $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

$\Rightarrow$ Centre = (p,q) = (2, - 3)

Here $a^2>b^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{9-\frac{9}{4}}{9}}$

$\Rightarrow e = \sqrt{\frac{\frac{27}{4}}{9}}$

$\Rightarrow e = \sqrt{\frac{3}{4}}$

$\Rightarrow e = \frac{\sqrt{3}}{2}$

Length of the major axis 2a = 2(3) = 6

Length of the minor axis 2b = $2\left(\frac{3}{2}\right)$ = 3

$\Rightarrow$ Foci = (p±ae,q)

$\Rightarrow$ Foci = $\left(\left(2 \pm \left(3 \times \frac{\sqrt{3}}{2}\right)\right), -3\right)$

$\Rightarrow$ Foci = $\left(\left(2 \pm \frac{3\sqrt{3}}{2}\right), -3\right)$

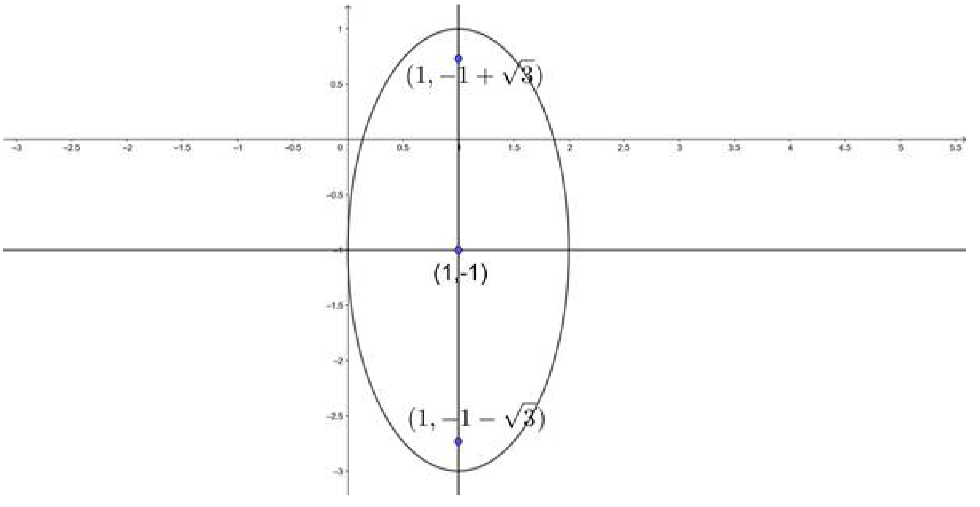
10 C. Question

Find the centre, the lengths of the axes, eccentricity, foci of the following ellipse:

$4x^2 + y^2 - 8x + 2y + 1 = 0$

Answer

Given that we need to find the centre, lengths of axes, eccentricity and foci of the ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$.



$\Rightarrow 4x^2 + y^2 - 8x + 2y + 1 = 0$

$\Rightarrow 4(x^2 - 2x + 1) + (y^2 + 2y + 1) - 4 = 0$

$\Rightarrow 4(x - 1)^2 + (y + 1)^2 = 4$

$\Rightarrow \frac{4(x-1)^2}{4} + \frac{(y+1)^2}{4} = 1$

$\Rightarrow \frac{(x-1)^2}{1} + \frac{(y+1)^2}{4} = 1$

Comparing with the standard form $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

$\Rightarrow$ Centre = (p,q) = (1, - 1)

Here $b^2 > a^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{b^2-a^2}{b^2}}$

$\Rightarrow e = \sqrt{\frac{4-1}{4}}$

$\Rightarrow e = \sqrt{\frac{3}{4}}$

$\Rightarrow e = \frac{\sqrt{3}}{2}$

Length of the major axis 2b = 2(2) = 4

Length of the minor axis 2a = 2(1) = 2

$\Rightarrow$ Foci = $(p, q \pm be)$

$\Rightarrow$ Foci = $\left(1, -1 \pm 2\left(\frac{\sqrt{3}}{2}\right)\right)$

$\Rightarrow$ Foci = $(1, -1 \pm \sqrt{3})$

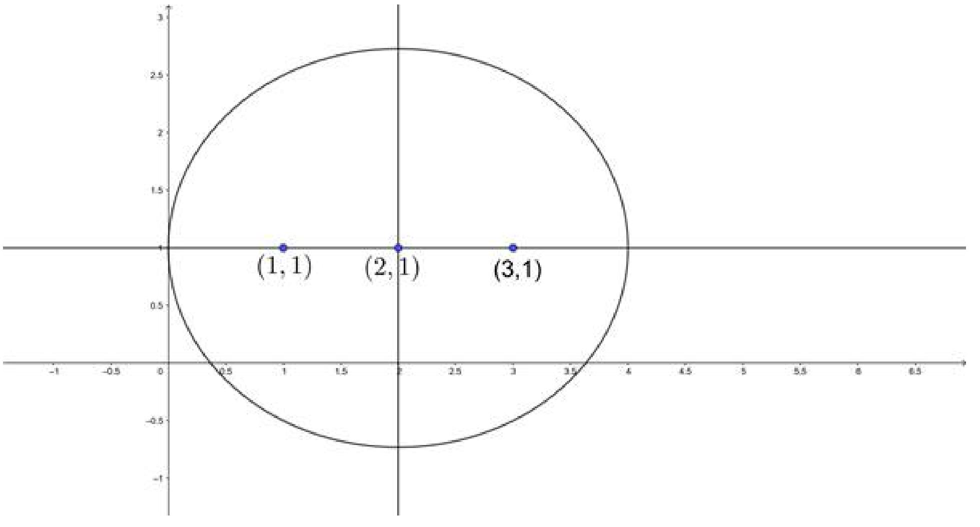
10 D. Question

Find the centre, the lengths of the axes, eccentricity, foci of the following ellipse:

$3x^2 + 4y^2 - 12x - 8y + 4 = 0$

Answer

Given that we need to find the centre, lengths of axes, eccentricity and foci of the ellipse $3x^2 + 4y^2 - 12x - 8y + 4 = 0$.



$\Rightarrow 3x^2 + 4y^2 - 12x - 8y + 4 = 0$

$\Rightarrow 3(x^2 - 4x + 4) + 4(y^2 - 2y + 1) - 12 = 0$

$\Rightarrow 3(x - 2)^2 + 4(y - 1)^2 = 12$

$\Rightarrow \frac{3(x-2)^2}{12} + \frac{4(y-1)^2}{12} = 1$

$\Rightarrow \frac{(x-2)^2}{4} + \frac{(y-1)^2}{3} = 1$

Comparing with the standard form $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

$\Rightarrow$ Centre = (p,q) = (2,1)

Here $a^2 > b^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{4-3}{4}}$

$\Rightarrow e = \sqrt{\frac{1}{4}}$

$\Rightarrow e = \frac{1}{2}$

Length of the major axis 2a = 2(2) = 4

Length of the minor axis 2b = $2(\sqrt{3}) = 2\sqrt{3}$

$\Rightarrow$ Foci = $(p \pm ae, q)$

$\Rightarrow$ Foci = $\left(\left(2 \pm \left(2 \times \frac{1}{2}\right)\right), 1\right)$

⇒ Foci = $((2 \pm 1), 1)$

⇒ Foci = (3,1) and (1,1)

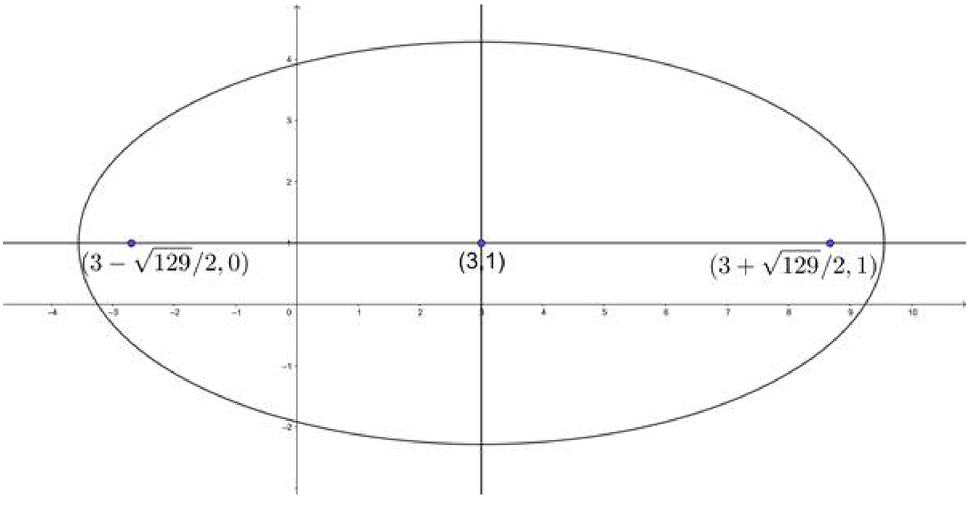
10 E. Question

Find the centre, the lengths of the axes, eccentricity, foci of the following ellipse:

$4x^2 + 16y^2 - 24x - 32y - 12 = 0$

Answer

Given that we need to find the centre, lengths of axes, eccentricity and foci of the ellipse $4x^2 + 16y^2 - 24x - 32y - 120 = 0$.



⇒ $4x^2 + 16y^2 - 24x - 32y - 120 = 0$

⇒ $4(x^2 - 6x + 9) + 16(y^2 - 2y + 1) - 172 = 0$

⇒ $4(x - 3)^2 + 16(y - 1)^2 = 172$

⇒ $\frac{4(x-3)^2}{172} + \frac{16(y-1)^2}{172} = 1$

⇒ $\frac{(x-3)^2}{43} + \frac{(y-1)^2}{\frac{43}{4}} = 1$

Comparing with the standard form $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

⇒ Centre = (p,q) = (3,1)

Here $a^2 > b^2$

⇒ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

⇒ $e = \sqrt{\frac{43-\frac{43}{4}}{43}}$

⇒ $e = \sqrt{\frac{\frac{3}{4}}{1}}$

⇒ $e = \sqrt{\frac{3}{4}}$

⇒ $e = \frac{\sqrt{3}}{2}$

Length of the major axis 2a = $2(\sqrt{43}) = 2\sqrt{43}$

Length of the minor axis 2b = $2\left(\frac{\sqrt{43}}{2}\right) = \sqrt{43}$

⇒ Foci = (p±ae,q)

⇒ Foci = $\left(\left(3 \pm \left(\sqrt{43} \times \frac{\sqrt{3}}{2}\right)\right), 1\right)$

⇒ Foci = $\left(\left(3 \pm \frac{\sqrt{129}}{2}\right), 1\right)$

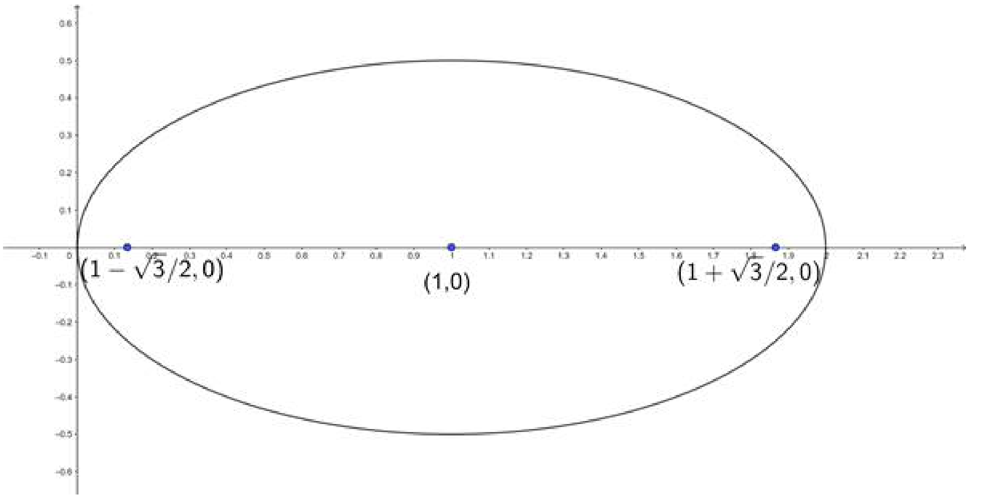
10 F. Question

Find the centre, the lengths of the axes, eccentricity, foci of the following ellipse:

$x^2 + 4y^2 - 2x = 0$

Answer

Given that we need to find the centre, lengths of axes, eccentricity and foci of the ellipse $x^2 + 4y^2 - 2x = 0$.



⇒ $x^2 + 4y^2 - 2x = 0$

⇒ $(x^2 - 2x + 1) + 4(y^2) - 1 = 0$

⇒ $(x - 1)^2 + 4(y - 0)^2 = 1$

⇒ $\frac{(x-1)^2}{1} + \frac{4(y-0)^2}{1} = 1$

⇒ $\frac{(x-1)^2}{1} + \frac{(y-0)^2}{\frac{1}{4}} = 1$

Comparing with the standard form $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

⇒ Centre = (p,q) = (1,0)

Here $a^2 > b^2$

⇒ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

⇒ $e = \sqrt{\frac{1-\frac{1}{4}}{1}}$

⇒ $e = \sqrt{\frac{3}{4}}$

⇒ $e = \frac{\sqrt{3}}{2}$

Length of the major axis 2a = 2(1) = 2

Length of the minor axis 2b = $2\left(\frac{1}{2}\right)$ = 1

⇒ Foci = (p±ae,q)

⇒ Foci = $\left(\left(1 \pm \left(1 \times \frac{\sqrt{3}}{2}\right)\right), 0\right)$

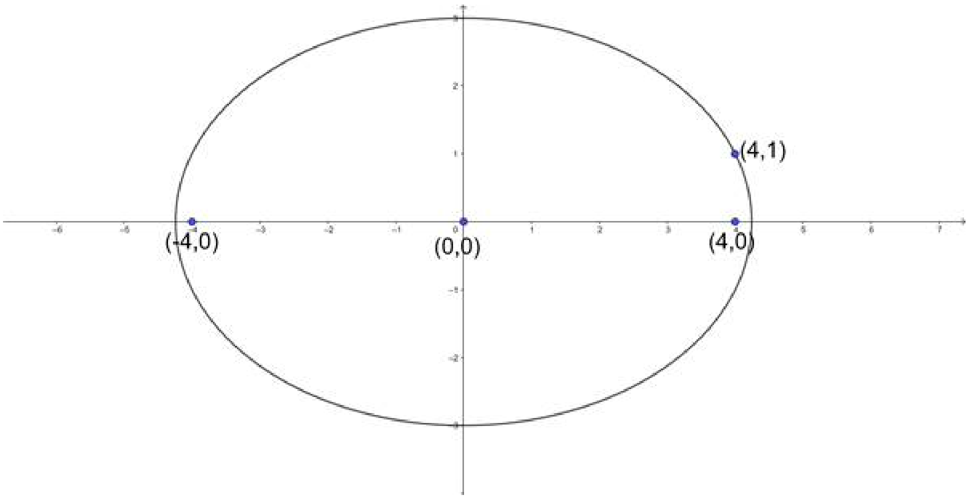
⇒ Foci = $\left(\left(1 \pm \frac{3}{\sqrt{2}}\right), 0\right)$

11. Question

Find the equation of an ellipse whose foci are at(± 3, 0) and which passes through (4, 1).

Answer

Given that we need to find the equation of the ellipse whose foci are at (±4,0) and passes through (4,1).



Let us assume the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ - - - - (1) ($a^2 > b^2$).

We know that foci are (±ae,0) and eccentricity of the ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$

⇒ ae = 3

⇒ $a\sqrt{\frac{a^2-b^2}{a^2}} = 3$

⇒ $a^2 - b^2 = 9$ ..... - - - (2)

Substituting the point (4,1) in (1) we get,

⇒ $\frac{4^2}{a^2} + \frac{1^2}{b^2} = 1$

⇒ $\frac{16}{a^2} + \frac{1}{b^2} = 1$

⇒ $16b^2 + a^2 = a^2b^2$

From (2),

⇒ $16(a^2 - 9) + a^2 = a^2(a^2 - 9)$

⇒ $16a^2 - 144 + a^2 = a^4 - 9a^2$

⇒ $a^4 - 26a^2 + 144 = 0$

⇒ $a^4 - 18a^2 - 8a^2 + 144 = 0$

$\Rightarrow a^2(a^2 - 18) - 8(a^2 - 18) = 0$

$\Rightarrow (a^2 - 8)(a^2 - 18) = 0$

$\Rightarrow a^2 - 8 = 0$ (or) $a^2 - 18 = 0$

$\Rightarrow a^2 = 8$ (or) $a^2 = 18$

$\Rightarrow b^2 = 18 - 9$(since $b^2>0$)

$\Rightarrow b^2 = 9.$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{18} + \frac{y^2}{9} = 1$

$\Rightarrow \frac{x^2 + 2y^2}{18} = 1$

$\Rightarrow x^2 + 2y^2 = 18$

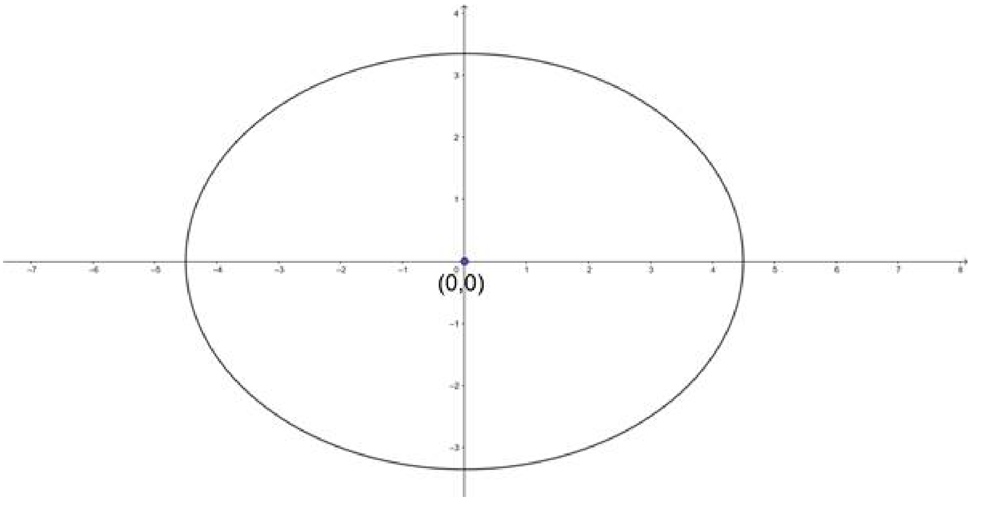
∴ The equation of the ellipse is $x^2 + 2y^2 = 18$.

12. Question

Find the equation of an ellipse whose eccentricity is 2/3, the latus - rectum is 5 and the centre is at the origin.

Answer

Given that we need to find the equation of the ellipse whose eccentricity is $\frac{2}{3}$, latus - rectum is 5 and centre is at origin.



Let us assume the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ - - - - (1) ($a^2>b^2$) since centre is at origin.

We know that eccentricity of the ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \sqrt{\frac{a^2-b^2}{a^2}} = \frac{2}{3}$

$\Rightarrow \frac{a^2-b^2}{a^2} = \frac{4}{9}$

$\Rightarrow 9(a^2 - b^2) = 4a^2$

$\Rightarrow 5a^2 = 9b^2$

$\Rightarrow b^2 = \frac{5a^2}{9}$..... - - - (2)

We know that length of the latus - rectum is $\frac{2b^2}{a}$

$\Rightarrow \frac{2b^2}{a} = 5$

$\Rightarrow \frac{2\left(\frac{5a^2}{9}\right)}{a} = 5$

$\Rightarrow \frac{10a}{9} = 5$

$\Rightarrow a = \frac{9}{2}$

$\Rightarrow a^2 = \frac{81}{4}$

From (2),

$\Rightarrow b^2 = \frac{5\left(\frac{81}{4}\right)}{9}$

$\Rightarrow b^2 = \frac{45}{4}$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$

$\Rightarrow \frac{4x^2}{81} + \frac{4y^2}{45} = 1$

$\Rightarrow \frac{20x^2 + 36y^2}{405} = 1$

$\Rightarrow 20x^2 + 36y^2 = 405$

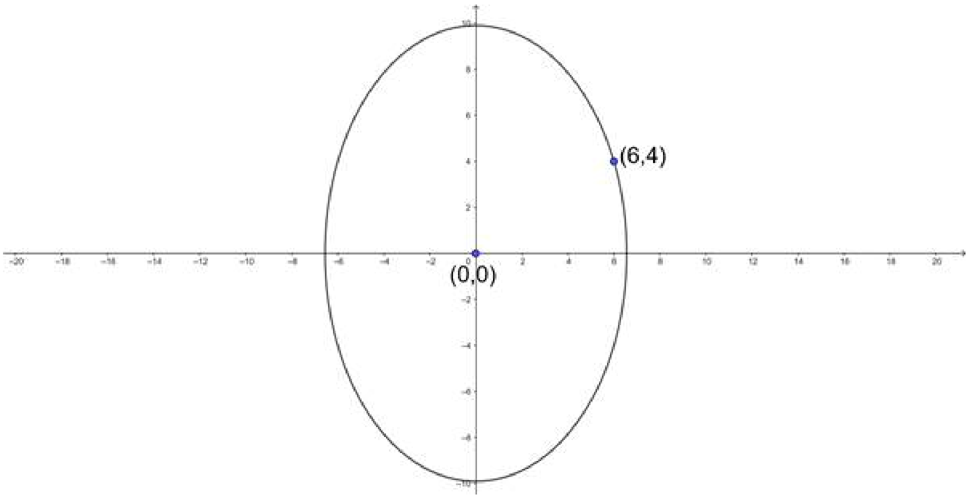
$\therefore$ The equation of the ellipse is $20x^2 + 36y^2 = 405$.

13. Question

Find the equation of an ellipse with its foci on y - axis, eccentricity 3/4, centre at the origin and passing through (6, 4).

Answer

Given that we need to find the equation of the ellipse whose eccentricity is $\frac{3}{4}$, centre at the origin and passes through (6,4).



Let us assume the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ - - - - (1) ($a^2<b^2$), since centre is at origin and foci on y - axis.

We know that eccentricity of the ellipse is $e = \sqrt{\frac{b^2-a^2}{b^2}}$

$\Rightarrow \sqrt{\frac{b^2-a^2}{b^2}} = \frac{3}{4}$

$\Rightarrow \frac{b^2-a^2}{b^2} = \frac{9}{16}$

$\Rightarrow 16b^2 - 16a^2 = 9b^2$

$\Rightarrow 7b^2 = 16a^2$

$\Rightarrow a^2 = \frac{7b^2}{16}$ ..... - - - (2)

Substituting the point (6,4) in (1) we get,

$\Rightarrow \frac{6^2}{a^2} + \frac{4^2}{b^2} = 1$

$\Rightarrow \frac{36}{\frac{7b^2}{16}} + \frac{16}{b^2} = 1$

$\Rightarrow \frac{576}{7b^2} + \frac{16}{b^2} = 1$

$\Rightarrow \frac{576+112}{7b^2} = 1$

$\Rightarrow 7b^2 = 688$

$\Rightarrow b^2 = \frac{688}{7}$

From (2),

$\Rightarrow a^2 = \frac{7\left(\frac{688}{7}\right)}{16}$

$\Rightarrow a^2 = \frac{688}{16}$

$\Rightarrow a^2 = 43$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{43} + \frac{y^2}{\frac{688}{7}} = 1$

$\Rightarrow \frac{x^2}{43} + \frac{7y^2}{688} = 1$

$\Rightarrow \frac{16x^2+7y^2}{688} = 1$

$\Rightarrow 16x^2 + 7y^2 = 688$

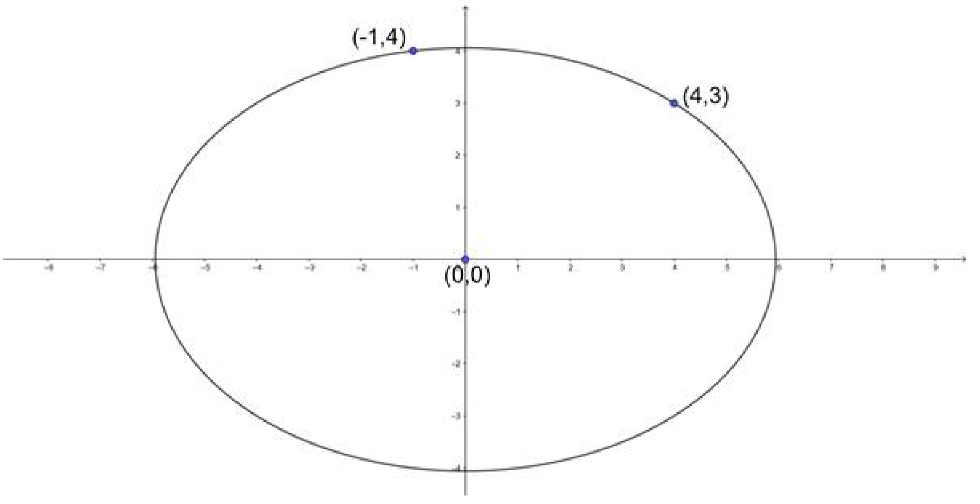
$\therefore$ The equation of the ellipse is $16x^2 + 7y^2 = 688$.

14. Question

Find the equation of an ellipse whose axes lie along coordinates axes and which passes through (4, 3) and (- 1, 4).

Answer

Given that we need to find the equation of the ellipse passing through the points (4,3) and (- 1,4).



Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2 > b^2$). - - - - (1)

Substituting the point (4,3) in (1) we get

$\Rightarrow \frac{4^2}{a^2} + \frac{3^2}{b^2} = 1$

$\Rightarrow \frac{16}{a^2} + \frac{9}{b^2} = 1$

$\Rightarrow \frac{16b^2 + 9a^2}{a^2b^2} = 1$

$\Rightarrow 16b^2 + 9a^2 = a^2\ b^2$ ..... - - (2)

Substituting the point (- 1,4) in (1) we get

$\Rightarrow \frac{(-1)^2}{a^2} + \frac{4^2}{b^2} = 1$

$\Rightarrow \frac{1}{a^2} + \frac{16}{b^2} = 1$

$\Rightarrow \frac{b^2 + 16a^2}{a^2b^2} = 1$

$\Rightarrow b^2 + 16a^2 = a^2b^2$ ..... - - (3)

(3)×16 - (2)

$\Rightarrow (16b^2 + 256a^2) - (9a^2 + 16b^2) = (16a^2b^2 - a^2b^2)$

$\Rightarrow 247a^2 = 15a^2b^2$

$\Rightarrow 15b^2 = 247$

$\Rightarrow b^2 = \frac{247}{15}$

From (3)

$\Rightarrow \frac{247}{15} + 16a^2 = a^2\left(\frac{247}{15}\right)$

$\Rightarrow a^2\left(\frac{247-240}{15}\right) = \frac{247}{15}$

$\Rightarrow a^2\left(\frac{7}{15}\right) = \frac{247}{15}$

$\Rightarrow a^2 = \frac{247}{7}$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{\frac{247}{7}} + \frac{y^2}{\frac{247}{15}} = 1$

$\Rightarrow \frac{7x^2}{247} + \frac{15y^2}{247} = 1$

$\Rightarrow 7x^2 + 15y^2 = 247$

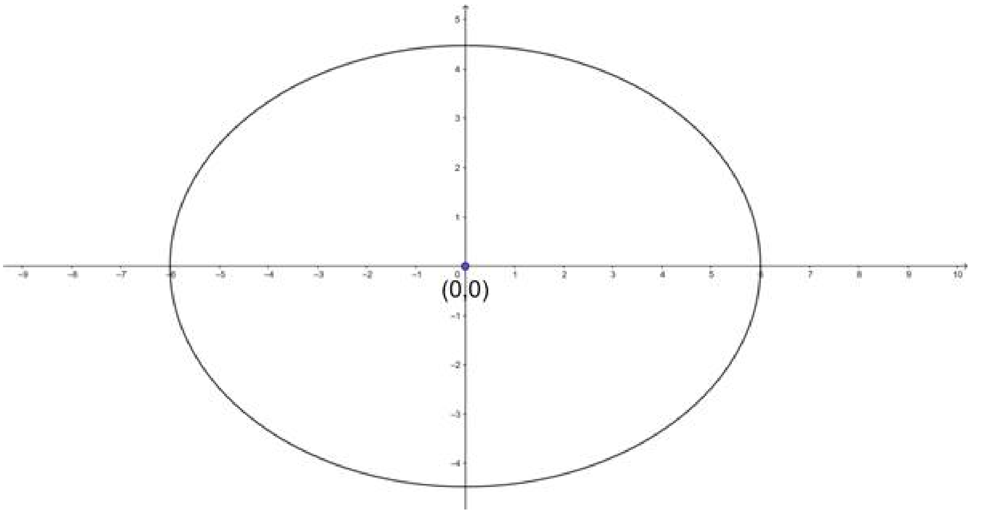
$\therefore$ The equation of the ellipse is $7x^2 + 15y^2 = 247$.

15. Question

Find the equation of an ellipse whose axes lie along the coordinates axes, which passes through the point (- 3, 1) and has eccentricity equal to $\sqrt{2/5}$.

Answer

Given that we need to find the equation of the ellipse whose eccentricity is $\sqrt{\frac{2}{5}}$ and passes through (- 3,1).



Let us assume the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ - - - - (1) ($a^2>b^2$).

We know that eccentricity of the ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{\frac{2}{5}}$

$\Rightarrow \frac{a^2-b^2}{a^2} = \frac{2}{5}$

$\Rightarrow 5a^2 - 5b^2 = 2a^2$

$\Rightarrow 5b^2 = 3a^2$

$\Rightarrow a^2 = \frac{5b^2}{3}$ ..... - - - (2)

Substituting the point (- 3,1) in (1) we get,

$\Rightarrow \frac{(-3)^2}{a^2} + \frac{1^2}{b^2} = 1$

$\Rightarrow \frac{9}{\frac{5b^2}{3}} + \frac{1}{b^2} = 1$

$\Rightarrow \frac{27}{5b^2} + \frac{1}{b^2} = 1$

$\Rightarrow \frac{27+5}{5b^2} = 1$

$\Rightarrow 5b^2 = 32$

$\Rightarrow b^2 = \frac{32}{5}$

From (2),

$\Rightarrow a^2 = \frac{5\left(\frac{32}{5}\right)}{3}$

$\Rightarrow a^2 = \frac{32}{3}$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$

$\Rightarrow \frac{3x^2}{32} + \frac{5y^2}{32} = 1$

$\Rightarrow \frac{3x^2 + 5y^2}{32} = 1$

$\Rightarrow 3x^2 + 5y^2 = 32$

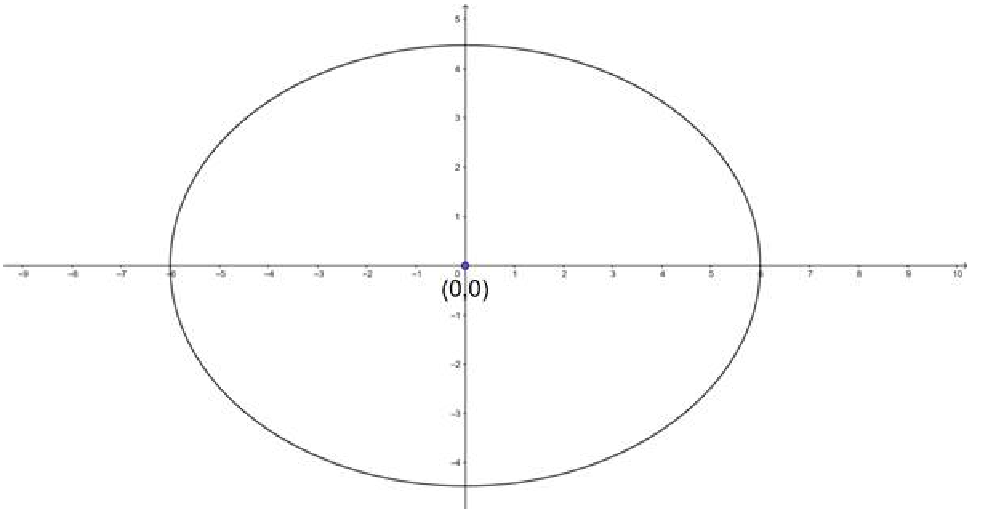
$\therefore$ The equation of the ellipse is $3x^2 + 5y^2 = 32$.

16. Question

Find the equation of an ellipse, the distance between the foci is 8 units and the distance between the directrices is 18 units.

Answer

Given that we need to find the equation of the ellipse whose distance between the foci is 8 units and distance between the directrices is 18 units.



We know that the distance between the foci is 2ae.

$\Rightarrow 2ae = 8$

$\Rightarrow ae = 4$ ..... - (1)

We know that the distance between the directrices is $\frac{2a}{e}$.

$\Rightarrow \frac{2a}{e} = 18$

$\Rightarrow \frac{a}{e} = 9$ ..... - (2)

(1)×(2)

$\Rightarrow ae \times \frac{a}{e} = 4 \times 9$

$\Rightarrow a^2 = 36$ .... (3)

$\Rightarrow a = 6$

We know that eccentricity of an ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow a\sqrt{\frac{a^2-b^2}{a^2}} = 4$

$\Rightarrow 36 - b^2 = 16$

$\Rightarrow b^2 = 20$ ..... (4)

The equation of the ellipse is,

$\Rightarrow \frac{x^2}{36} + \frac{y^2}{20} = 1$

$\Rightarrow \frac{5x^2+9y^2}{180} = 1$

$\Rightarrow 5x^2 + 9y^2 = 180$

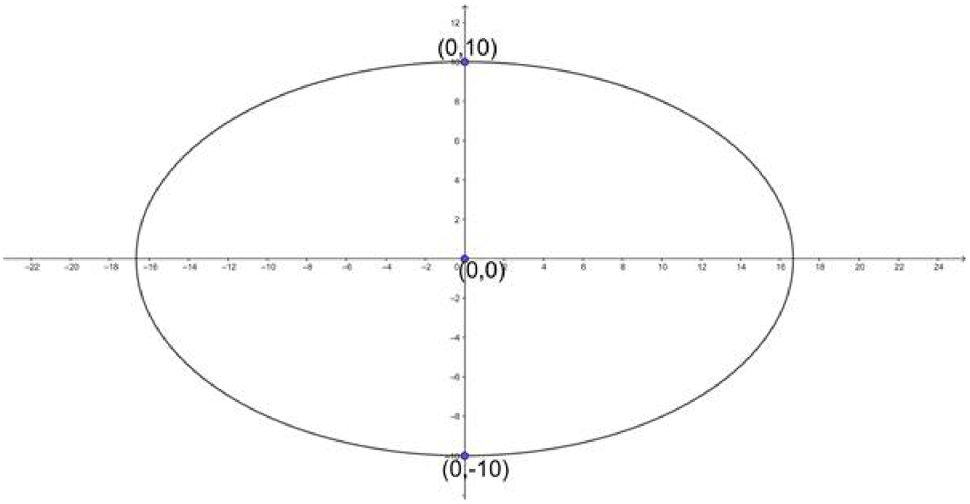
$\therefore$ The equation of the ellipse is $5x^2 + 9y^2 = 180$.

17. Question

Find the equation of an ellipse whose vertices are (0, ± 10) and eccentricity $e = \frac{4}{5}$.

Answer

Given that we need to find the equation of the ellipse whose vertices are (0,±10) and eccentricity $e = \frac{4}{5}$.



Let us assume the equation of the ellipse as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ - - - - (1) ($a^2 > b^2$).

We know that vertices of the ellipse are (0,±b)

$\Rightarrow b = 10$

$\Rightarrow b^2 = 100$

We know that eccentricity $e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow \frac{4}{5} = \sqrt{\frac{a^2-100}{a^2}}$

$\Rightarrow \frac{16}{25} = 1 - \frac{100}{a^2}$

$\Rightarrow \frac{100}{a^2} = \frac{9}{25}$

$\Rightarrow a^2 = \frac{2500}{9}$

The equation of the ellipse is

$\Rightarrow \frac{x^2}{\frac{2500}{9}} + \frac{y^2}{100} = 1$

$\Rightarrow \frac{9x^2}{2500} + \frac{y^2}{100} = 1$

$\Rightarrow \frac{9x^2 + 25y^2}{2500} = 1$

$\Rightarrow 9x^2 + 25y^2 = 2500$

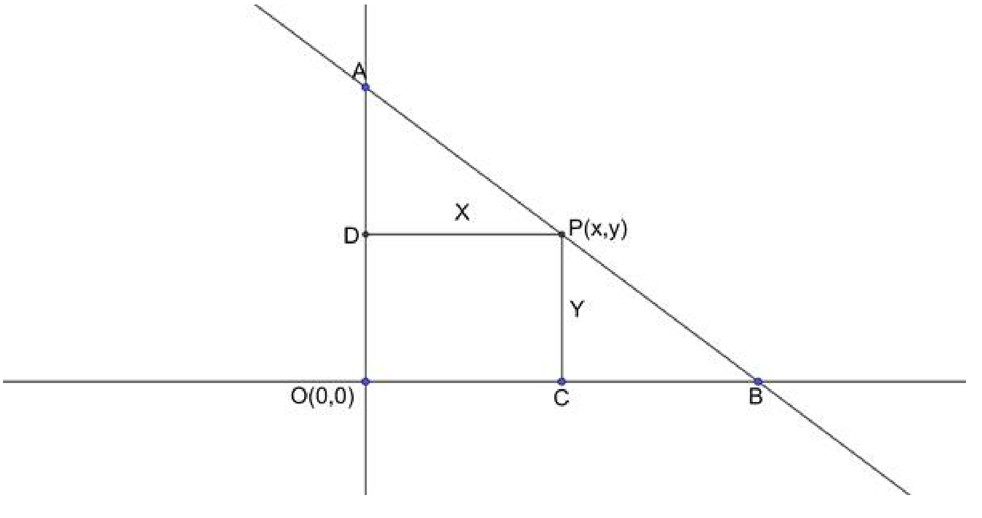
$\therefore$ The equation of the ellipse is $9x^2 + 25y^2 = 2500$.

18. Question

A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with x - axis.

Answer

Given that we need to find the locus of the point on the rod whose ends always touching the coordinate axes.



We need to the equation of locus of point P on the rod, which is 3 cm from the end in contact with x - axis.

Let us assume AB be the rod of length 12 cm and P(x,y) be the required point.

From the figure using similar triangles DAP and CBP we get,

$\Rightarrow \frac{AD}{PC} = \frac{AP}{PB}$

$\Rightarrow \frac{q}{y} = \frac{9}{3}$

$\Rightarrow q = 3y$ ..... (1)

$\Rightarrow \frac{DP}{CB} = \frac{AP}{PB}$

$\Rightarrow \frac{x}{p} = \frac{9}{3}$

$\Rightarrow p = \frac{x}{3}$ ..... - (2)

Now OB = OC + CB

$\Rightarrow OB = x + \frac{x}{3}$

$\Rightarrow OB = \frac{4x}{3}$ .... (3)

$\Rightarrow$ OA = OD + DA

$\Rightarrow$ OA = y + 3y

$\Rightarrow$ OA = 4y .... (4)

Since OAB is a right angled triangle,

$\Rightarrow OA^2 + OB^2 = AB^2$

$\Rightarrow (4y)^2 + \left(\frac{4x}{3}\right)^2 = (12)^2$

$\Rightarrow 16y^2 + \frac{16x^2}{9} = 144$

$\Rightarrow y^2 + \frac{x^2}{9} = 9$

$\Rightarrow \frac{9y^2 + x^2}{9} = 9$

$\Rightarrow x^2 + 9y^2 = 81$

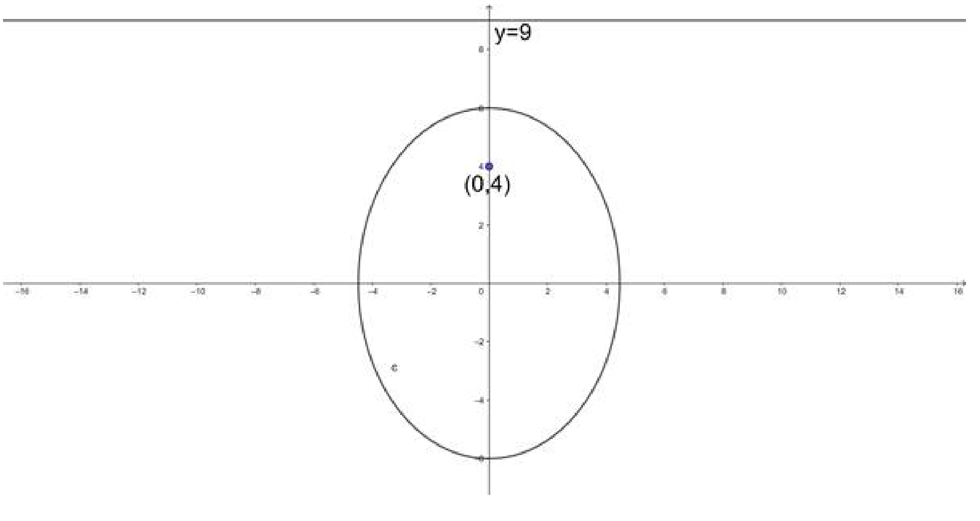
$\therefore$ The equation of the ellipse is $x^2 + 9y^2 = 81$.

19. Question

Find the equation of the set of all points whose distances from (0, 4) are $\frac{2}{3}$ of their distances from the line y = 9.

Answer

Given that we need to find the equation of set of all points whose distances from S(0,4) are $\frac{2}{3}$ of their distances from the line(M) y = 9.



Let P(x,y) be any point from the set of all points.

We know that distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

$\Rightarrow SP = \left(\frac{2}{3}\right) PM$

$\Rightarrow SP^2 = \frac{4}{9} PM^2$

$\Rightarrow (x-0)^2 + (y-4)^2 = \frac{4}{9}\left(\frac{|y-9|}{\sqrt{1^2}}\right)^2$

$\Rightarrow x^2 + y^2 - 8y + 16 = \frac{4}{9} \times \frac{(|y-9|)^2}{1}$

$\Rightarrow x^2 + y^2 - 8y + 16 = \frac{4}{9} \times (y^2 - 18y + 81)$

$\Rightarrow 9x^2 + 9y^2 - 72y + 144 = 4y^2 - 72y + 324$

$\Rightarrow 9x^2 + 5y^2 = 180$

∴ The equation of the ellipse is $9x^2 + 5y^2 = 180$.

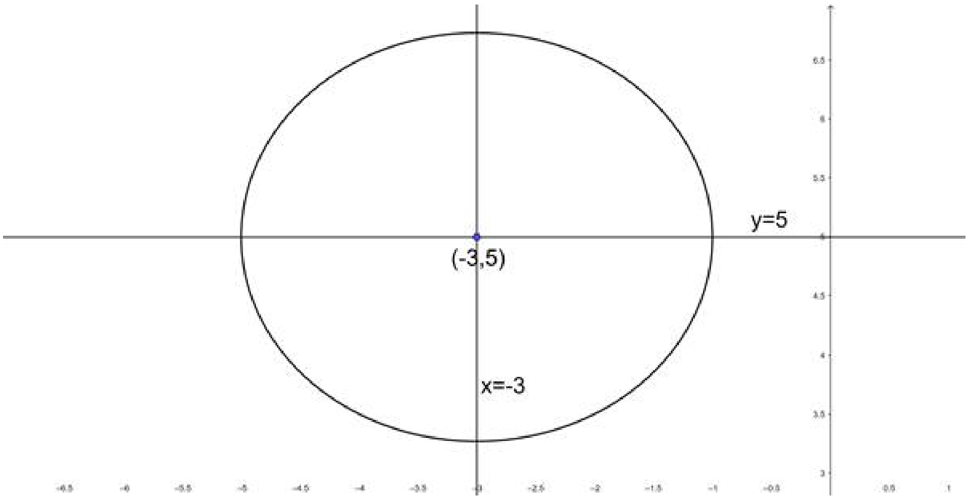
Very Short Answer

1. Question

If the lengths of semi - major and semi - minor axes of an ellipse are 2 and √3 and their corresponding equations are y – 5 = 0 and x + 3 = 0, then write the equation of the ellipse.

Answer

Given that we need to find the equation of the ellipse whose semi - major and semi - minor axes are 2 and $\sqrt{3}$ and their corresponding equations of equations of axes are y - 5 = 0 and x + 3 = 0.



We know that the centre is the point of intersection of both axes. So, on solving these axes we get the centre to be (- 3,5).

$\Rightarrow$ Semi - major axis(a) = 2

$\Rightarrow a^2 = 4$

$\Rightarrow$ Semi - minor axis(b) = $\sqrt{3}$

$\Rightarrow b^2 = 3$

We know that the equation of the ellipse whose centre is (p,q) and the length of semi - major axis is a and semi - minor axis is b is $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$.

∴ The equation of the ellipse is $\frac{(x+3)^2}{4} + \frac{(y-5)^2}{3} = 1$.

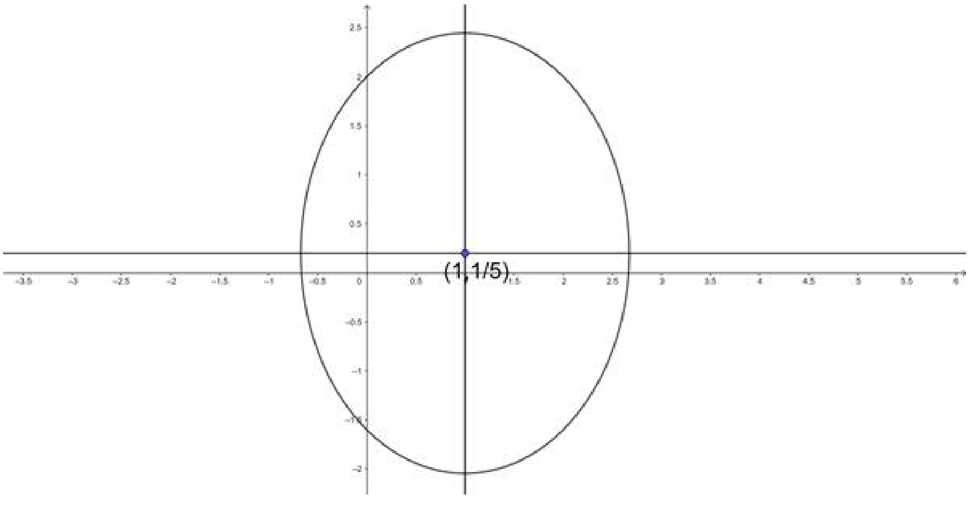
2. Question

Write the eccentricity of the ellipse $9x^2 + 5y^2 - 18x - 2y - 16 = 0$

Answer

Given the equation of the ellipse is $9x^2 + 5y^2 - 18x - 2y - 16 = 0$.

We need to find the eccentricity.



Given equation can be rewritten as

$$\Rightarrow 9(x^2 - 2x + 1) + 5\left(y^2 - \frac{2}{5}y + \frac{1}{25}\right) = \frac{126}{5}$$

$$\Rightarrow 9(x-1)^2 + 5\left(y - \frac{1}{5}\right)^2 = \frac{126}{5}$$

$$\Rightarrow \frac{9(x-1)^2}{\frac{126}{5}} + \frac{5\left(y-\frac{1}{5}\right)^2}{\frac{126}{5}} = 1$$

$$\Rightarrow \frac{(x-1)^2}{\frac{126}{45}} + \frac{\left(y-\frac{1}{5}\right)^2}{\frac{126}{25}} = 1.$$

We know for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($b^2 > a^2$)

$$\Rightarrow e = \sqrt{\frac{b^2 - a^2}{b^2}}$$

Here $a^2 = \frac{126}{45}$ and $b^2 = \frac{126}{25}$, $b^2 > a^2$

$$\Rightarrow e = \sqrt{\frac{\frac{126}{25} - \frac{126}{45}}{\frac{126}{25}}}$$

$$\Rightarrow e = \sqrt{\frac{\frac{4}{225}}{\frac{126}{25}}}$$

$$\Rightarrow e = \sqrt{\frac{2}{567}}$$

$$\Rightarrow e = \frac{\sqrt{14}}{63}$$

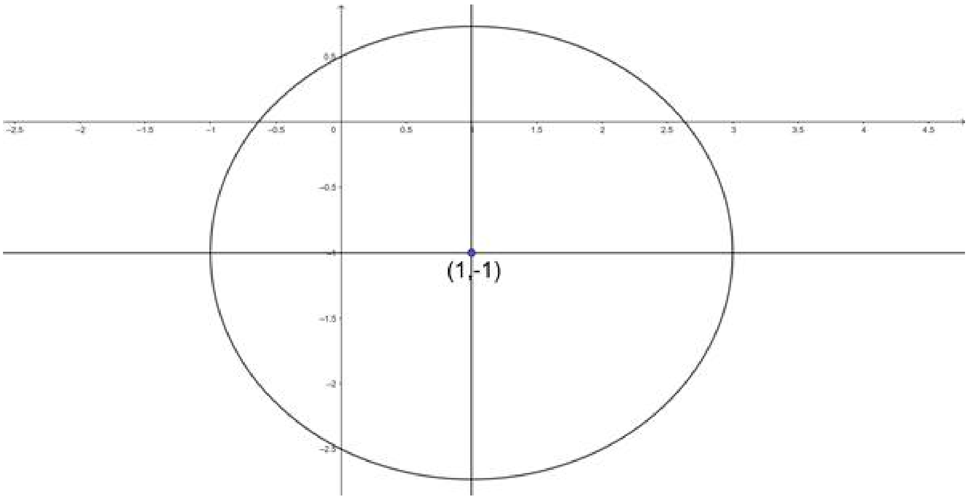
∴ The eccentricity is $\frac{\sqrt{14}}{63}$.

3. Question

Write the centre and eccentricity of the ellipse $3x^2 + 4y^2 - 6x + 8y - 5 = 0$

Answer

Given that we need to find the centre and eccentricity of the ellipse $3x^2 + 4y^2 - 6x + 8y - 5 = 0$.



$\Rightarrow 3x^2 + 4y^2 - 6x + 8y - 5 = 0$

$\Rightarrow 3(x^2 - 2x + 1) + 4(y^2 + 2y + 1) = 12$

$\Rightarrow 3(x - 1)^2 + 4(y + 1)^2 = 12$

$\Rightarrow \frac{3(x-1)^2}{12} + \frac{4(y+1)^2}{12} = 1$

$\Rightarrow \frac{(x-1)^2}{4} + \frac{(y+1)^2}{3} = 1$

Comparing with the standard form $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

$\Rightarrow$ Centre = (p,q) = (1, - 1)

Here $a^2>b^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{4-3}{4}}$

$\Rightarrow e = \sqrt{\frac{1}{4}}$

$\Rightarrow e = \frac{1}{2}$

4. Question

PSQ is focal chord of the ellipse $4x^2 + 9y^2 = 36$ such that SP = 4. If S' is the another focus, write the value of S'Q.

Answer

Given that PSQ is the focal chord of the ellipse $4x^2 + 9y^2 = 36$.

It is also given that SP = 4. We need to find the value of S'Q, where S and S' are foci.

Given ellipse is rewritten as $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

We got $a^2 = 9$, a = 3 and $b^2 = 4$.

We know that semi latus rectum is the harmonic mean of any two segments of focal chord.

$\Rightarrow \frac{2}{\frac{b^2}{a}} = \frac{1}{SP} + \frac{1}{SQ}$

$\Rightarrow \frac{2\times3}{4} = \frac{1}{4} + \frac{1}{SQ}$

$\Rightarrow \frac{5}{4} = \frac{1}{SQ}$

$\Rightarrow SQ = \frac{4}{5}$

We know that SQ + S’Q = 2a

$\Rightarrow \frac{4}{5} + S'Q = 2(3)$

$\Rightarrow S'Q = 6 - \frac{4}{5}$

$\Rightarrow S'Q = \frac{26}{5}$

5. Question

Write the eccentricity of an ellipse whose latus - rectum is one half of the minor axis.

Answer

Given that we need to find the eccentricity of the ellipse whose latus - rectum is one half of the minor axis.

We know that length of latus rectum is $\frac{2b^2}{a}$ and length of minor axis is 2b.

$\Rightarrow \frac{2b^2}{a} = b$

⇒ a = 2b.

We know that eccentricity of the ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{(2b)^2-b^2}{(2b)^2}}$

$\Rightarrow e = \sqrt{\frac{3b^2}{4b^2}}$

$\Rightarrow e = \frac{\sqrt{3}}{2}.$

6. Question

If the distance between the foci of an ellipse is equal to the length of the latus - rectum, write the eccentricity of the ellipse.

Answer

Given that we need to find the eccentricity of the ellipse whose latus - rectum is equal to the distance between the foci.

We know that length of latus rectum is $\frac{2b^2}{a}$ and distance between foci is 2ae.

$\Rightarrow \frac{2b^2}{a} = 2ae$

⇒ $b^2 = a^2e$.

We know that eccentricity of the ellipse is $b^2 = a^2(1 - e^2)$

⇒ $a^2(1 - e^2) = a^2e$

⇒ $e^2 + e - 1 = 0$

$\Rightarrow e = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$

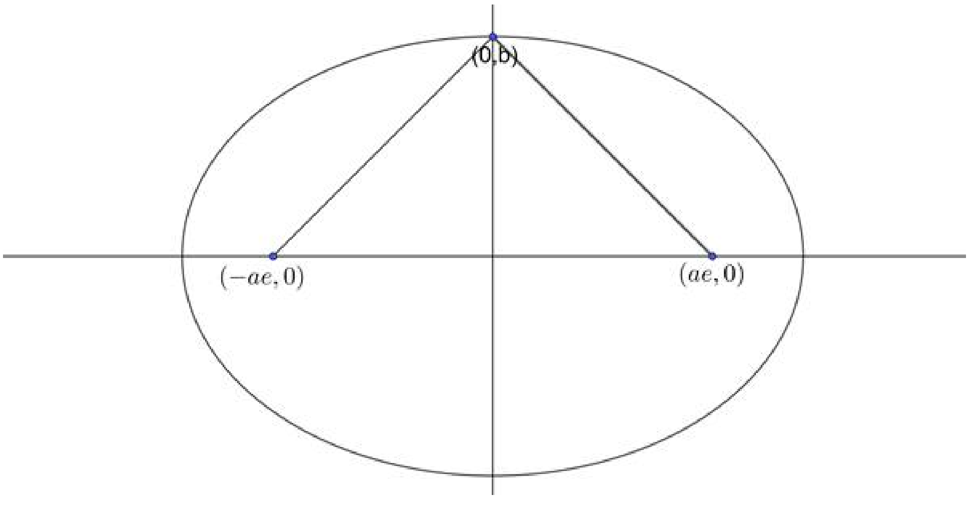
$\Rightarrow e = \frac{\sqrt{5}-1}{2}$(since e>0)

7. Question

If S and S' are two foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and B is an end of the minor axis such that Δ BSS' is equilateral, then write the eccentricity of the ellipse.

Answer

Given that S and S' are the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.



It is told that ΔBSS' is equilateral, where B is the end of the minor axis. We need to find the eccentricity of the ellipse.

Let us assume that B = (0,b)

We know that foci of the ellipse are (±ae,0).

We know that the distance between the foci is 2ae.

Let us find the distance SB

We know that the distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow SB = \sqrt{(ae - 0)^2 + (0 - b)^2}$

$\Rightarrow SB = \sqrt{a^2 e^2 + b^2}$.

We know that sides of an equilateral triangle are equal.

$\Rightarrow SB = 2ae$

$\Rightarrow SB^2 = 4a^2e^2$

$\Rightarrow a^2e^2 + b^2 = 4a^2e^2$

We know that $b^2 = a^2(1 - e^2)$,

$\Rightarrow a^2e^2 + a^2 - a^2e^2 = 4a^2e^2$

$\Rightarrow a^2 = 4a^2e^2$

$\Rightarrow e^2 = \frac{1}{4}$

$\Rightarrow e = \sqrt{\frac{1}{4}}$

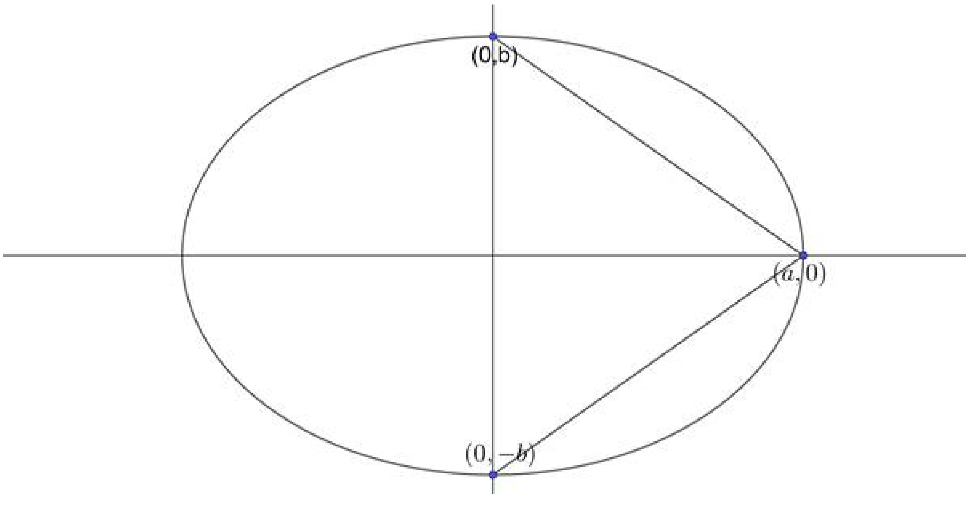
$\Rightarrow e = \frac{1}{2}.$

8. Question

If the minor axis of an ellipse subtends an equilateral triangle with vertex at one end of major axis, then write the eccentricity of the ellipse.

Answer

Given that the minor axis of an ellipse subtends an equilateral triangle with vertex at one end of major axis. We need to find the eccentricity of the ellipse.



Let us assume that ends of minor axis be B = (0,b) and C(0, - b) and end of major axis be A(a,0)

We know that the distance between the ends of minor axis is 2b.

Let us find the distance AB

We know that the distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow AB = \sqrt{(a - 0)^2 + (0 - b)^2}$

$\Rightarrow AB = \sqrt{a^2 + b^2}.$

We know that sides of an equilateral triangle are equal.

$\Rightarrow AB = 2b$

$\Rightarrow AB^2 = 4b^2$

$\Rightarrow a^2 + b^2 = 4b^2$

We know that $b^2 = a^2(1 - e^2)$,

$\Rightarrow a^2 + a^2 - a^2e^2 = 4a^2 - 4a^2e^2$

$\Rightarrow 2a^2 = 3a^2e^2$

$\Rightarrow e^2 = \frac{2}{3}$

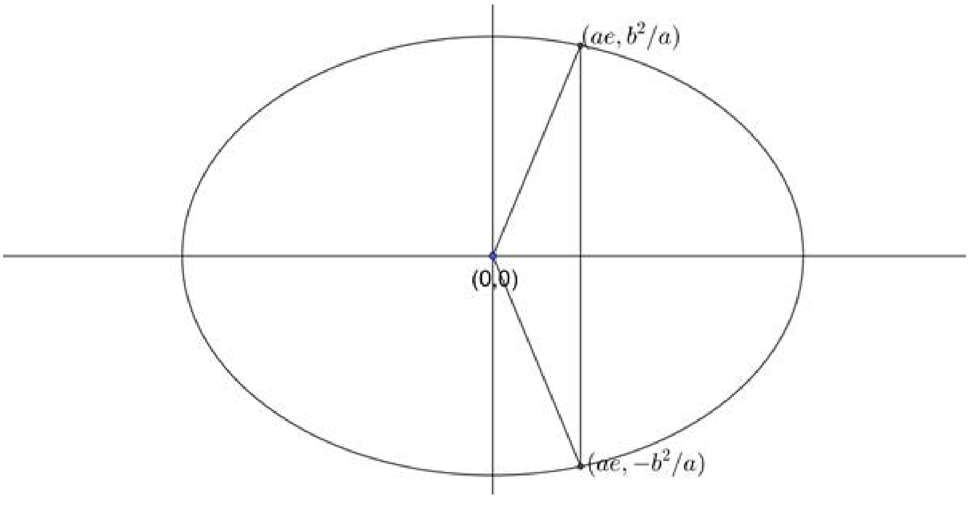
$\Rightarrow e = \sqrt{\frac{2}{3}}.$

9. Question

If a latus - rectum of an ellipse subtends a right angle at the centre of the ellipse, then write the eccentricity of the ellipse.

Answer

Given that the latus rectum of an ellipse subtends an right angle with centre of the ellipse. We need to find the eccentricity of the ellipse.



Let us assume that the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a^2>b^2$) such that the centre is O(0,0).

We know that the ends A and B of the latus rectum are $\left(ae, \pm\frac{b^2}{a}\right)$.

Let us find the slope(m_1) of the OA.

We know that the slope of the line joining points (x_1,y_1) and (x_2,y_2) is $\frac{y_2-y_1}{x_2-x_1}$

$\Rightarrow m_1 = \frac{\frac{b^2}{a}-0}{ae-0}$

$\Rightarrow m_1 = \frac{b^2}{a^2 e}$

Let us find the slope(m_2) of the OB.

We know that the slope of the line joining points (x_1,y_1) and (x_2,y_2) is $\frac{y_2-y_1}{x_2-x_1}$

$\Rightarrow m_2 = \frac{\frac{-b^2}{a}-0}{ae-0}$

$\Rightarrow m_2 = \frac{-b^2}{a^2 e}$

We know that the product of the slopes of the perpendicular is - 1.

$\Rightarrow m_1.m_2 = - 1$

$\Rightarrow \left(\frac{b^2}{a^2 e}\right).\left(\frac{-b^2}{a^2 e}\right) = - 1$

$\Rightarrow b^4 = a^4e^2$

$\Rightarrow e^2 = \frac{b^4}{a^4}$

$\Rightarrow e = \frac{b^2}{a^2}$

MCQ

1. Question

For the ellipse $12x^2 + 4y^2 + 24x – 16y + 25 = 0$

A. Centre is (- 1,2)

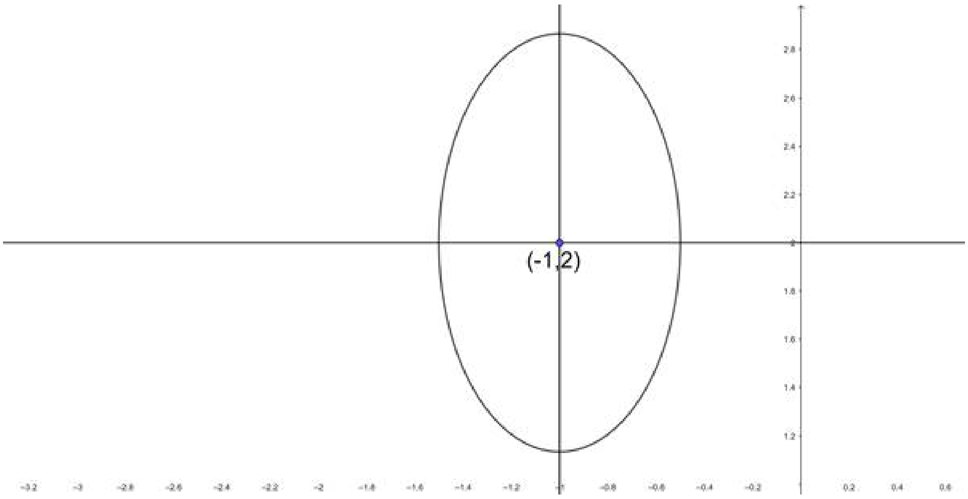
B. Lengths of the axes are $\sqrt{3}$ and 1

C. Eccentricity = $\sqrt{\frac{2}{3}}$

D. All of these

Answer

Given that we need to find the centre, lengths of axes, eccentricity and foci of the ellipse $12x^2 + 4y^2 + 24x - 16y + 25 = 0$.



$\Rightarrow 12x^2 + 4y^2 + 24x - 16y + 25 = 0$

$\Rightarrow 12(x^2 + 2x + 1) + 4(y^2 - 4y + 4) - 3 = 0$

$\Rightarrow 12(x + 1)^2 + 4(y - 2)^2 = 3$

$\Rightarrow \frac{12(x+1)^2}{3} + \frac{4(y-2)^2}{3} = 1$

$\Rightarrow \frac{(x+1)^2}{\frac{1}{4}} + \frac{(y-2)^2}{\frac{3}{4}} = 1$

Comparing with the standard form $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

$\Rightarrow$ Centre = (p,q) = (1, - 2)

Here $b^2>a^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{b^2-a^2}{b^2}}$

$\Rightarrow e = \sqrt{\frac{\frac{3}{4}-\frac{1}{4}}{\frac{3}{4}}}$

$\Rightarrow e = \sqrt{\frac{\frac{2}{4}}{\frac{3}{4}}}$

$\Rightarrow e = \sqrt{\frac{2}{3}}$

Length of the major axis 2b = $2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

Length of the minor axis 2a = $2\left(\frac{1}{2}\right)$ = 1

∴ The correct option is D

2. Question

The equation of ellipse with focus (- 1, 1), directrix x – y + 3 = 0 and eccentricity 1/2 is

A. $7x^2 + 2xy + 7y^2 + 10x + 10y + 7 = 0$

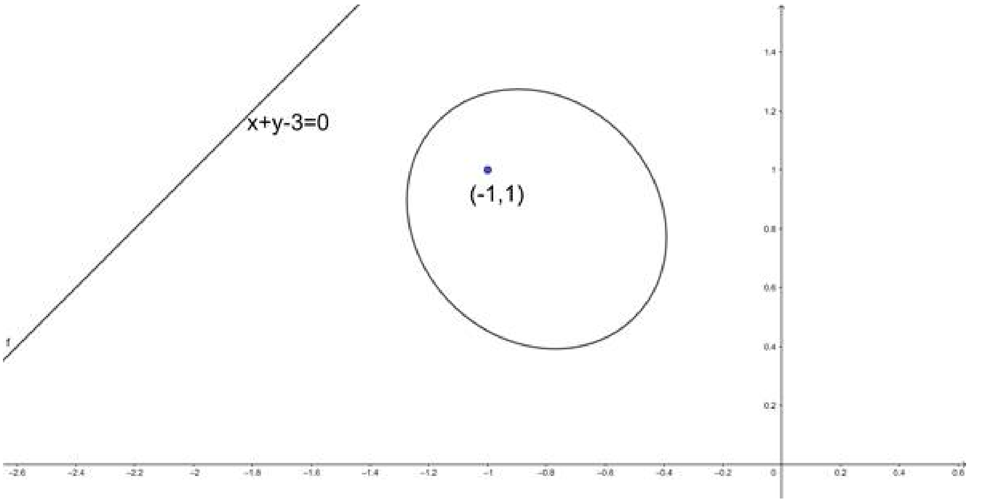
B. $7x^2 + 2xy + 7y^2 + 10x - 10y + 7 = 0$

C. $7x^2 + 2xy + 7y^2 + 10x - 10y - 7 = 0$

D. None of these

Answer

Given that we need to find the equation of the ellipse whose focus is S(- 1,1) and directrix(M) is x - y + 3 = 0 and eccentricity(e) is equal to $\frac{1}{2}$.



Let P(x,y) be any point on the ellipse.

We know that the distance between the focus and any point on ellipse is equal to the eccentricity times the perpendicular distance from that point to the directrix.

We know that distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

We know that the perpendicular distance from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$.

⇒ SP = ePM

⇒ $SP^2 = e^2PM^2$

⇒ $(x-(-1))^2 + (y-1)^2 = \left(\frac{1}{2}\right)^2\left(\frac{|x-y+3|}{\sqrt{1^2+(-1)^2}}\right)^2$

⇒ $x^2 + 2x + 1 + y^2 - 2y + 1 = \frac{1}{4}\times\frac{(|x-y+3|)^2}{1+1}$

⇒ $x^2 + y^2 + 2x - 2y + 2 = \frac{1}{8}\times(x^2 + y^2 + 9 - 2xy - 6y + 6x)$

⇒ $8x^2 + 8y^2 + 16x - 16y + 16 = x^2 + y^2 - 2xy - 6y + 6x + 9$

⇒ $7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$

∴ The correct option is B

3. Question

The equation of the circle drawn with the two foci of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ as the end - points of a diameter is

A. $x^2 + y^2 = a^2 + b^2$

B. $x^2 + y^2 = a^2$

C. $x^2 + y^2 = 2a^2$

D. $x^2 + y^2 = a^2 - b^2$

Answer

Given that we need to find the equation of the circle whose end points of diameter are the foci of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

We know that foci of ellipse are (±ae,0).

We know that (0,0) is the centre of the ellipse and midpoint of the foci.

We know that distance between centre and any focus is ae.

So, we have with centre at (0,0) and radius ae.

We know that eccentricity of the ellipse $e=\sqrt{\frac{a^2-b^2}{a^2}}$

We know that the equation of the circle whose centre is (p,q) and radius ‘r’ is $(x - p)^2 + (y - q)^2 = r^2$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = (ae)^2$

$\Rightarrow x^2 + y^2 = a^2e^2$

$\Rightarrow x^2+y^2=a^2\left(\frac{a^2-b^2}{a^2}\right)$

$\Rightarrow x^2 + y^2 = a^2 - b^2$

∴ The correct option is D

4. Question

The eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if its latus - rectum is equal to one half of its minor axis, is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{2}$

D. none of these

Answer

Given that we need to find the eccentricity of the ellipse whose latus - rectum is one half of the minor axis.

We know that length of latus rectum is $\frac{2b^2}{a}$ and length of minor axis is 2b.

$\Rightarrow \frac{2b^2}{a}=b$

$\Rightarrow$ a = 2b.

We know that eccentricity of the ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{(2b)^2-b^2}{(2b)^2}}$

$\Rightarrow e = \sqrt{\frac{3b^2}{4b^2}}$

$\Rightarrow e = \frac{\sqrt{3}}{2}$

∴ The correct option is A

5. Question

The eccentricity of the ellipse, if the distance between the foci is equal to the length of the latus - rectum is

A. $\frac{\sqrt{5}-1}{2}$

B. $\frac{\sqrt{5}+1}{2}$

C. $\frac{\sqrt{5}-1}{4}$

D. none of these

Answer

Given that we need to find the eccentricity of the ellipse whose latus - rectum is equal to the distance between the foci.

We know that length of latus rectum is $\frac{2b^2}{a}$ and distance between foci is 2ae.

$\Rightarrow \frac{2b^2}{a} = 2ae$

$\Rightarrow b^2 = a^2e$.

We know that eccentricity of the ellipse is $b^2 = a^2(1 - e^2)$

$\Rightarrow a^2(1 - e^2) = a^2e$

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \frac{-1\pm\sqrt{1^2-4(1)(-1)}}{2}$

$\Rightarrow e = \frac{\sqrt{5}-1}{2}$(since e>0)

∴ The correct option is A

6. Question

The eccentricity of the ellipse, if the minor axis is equal to the distance between the foci is

A. $\frac{\sqrt{3}}{2}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{\sqrt{2}}{3}$

Answer

Given that we need to find the eccentricity of the ellipse whose minor axis is equal to the distance between the foci.

We know that distance between foci is 2ae and length of minor axis is 2b.

⇒ 2ae = 2b

⇒ b = ae

⇒ $b^2 = a^2e^2$

We know that $b^2 = a^2(1 - e^2)$

⇒ $a^2(1 - e^2) = a^2e^2$

⇒ $a^2 = 2a^2e^2$

⇒ $e^2 = \frac{1}{2}$

⇒ $e = \frac{1}{\sqrt{2}}$

∴ The correct option is C

7. Question

The difference between the lengths of the major axis and the latus - rectum of an ellipse is

A. ae

B. 2ae

C. ae^2

D. $2ae^2$

Answer

Given that we need to find the difference between the lengths of major axis and length of latus rectum of an ellipse.

We know that the length of major axis is 2a and latus rectum is $\frac{2b^2}{a}$ for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$.

Let d be the difference.

⇒ $d = 2a - \frac{2b^2}{a}$

⇒ $d = \frac{2a^2 - 2b^2}{a}$

We know that $b^2 = a^2(1 - e^2)$

⇒ $d = \frac{2a^2 - 2a^2(1 - e^2)}{a}$

$\Rightarrow d = \frac{2a^2e^2}{a}$

$\Rightarrow d = 2ae^2$

∴ The correct option is D

8. Question

The eccentricity of the conic $9x^2 + 25y^2 = 225$ is

A. $\frac{2}{5}$

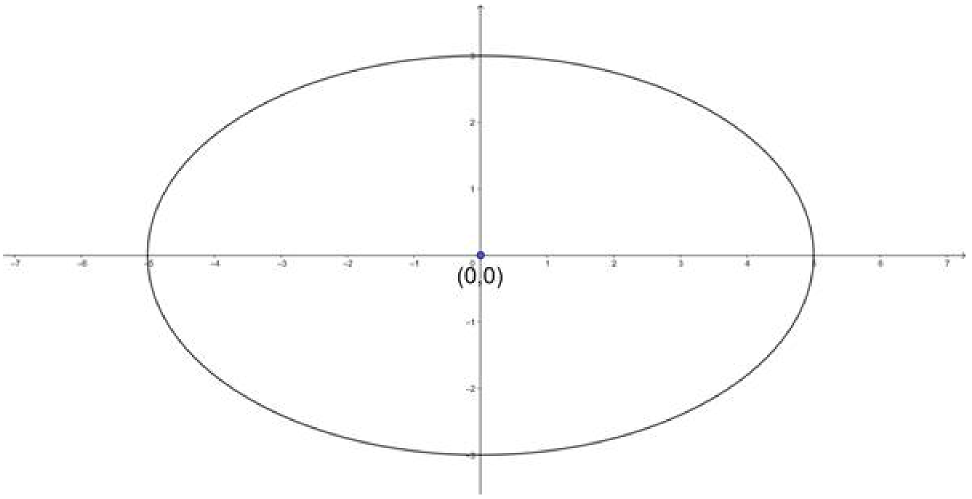
B. $\frac{4}{5}$

C. $\frac{1}{3}$

D. $\frac{3}{5}$

Answer

Given that we need to find the eccentricity of the conic $9x^2 + 25y^2 = 225$.



It is rewritten as $\frac{x^2}{25} + \frac{y^2}{9} = 1$

We know that the eccentricity of ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$ $(a^2>b^2)$.

$\Rightarrow e = \sqrt{\frac{25-9}{25}}$

$\Rightarrow e = \sqrt{\frac{16}{25}}$

$\Rightarrow e = \frac{4}{5}$

∴ The correct option is B

9. Question

The latus - rectum of the conic $3x^2 + 4y^2 - 6x + 8y - 5 = 0$ is

A. 3

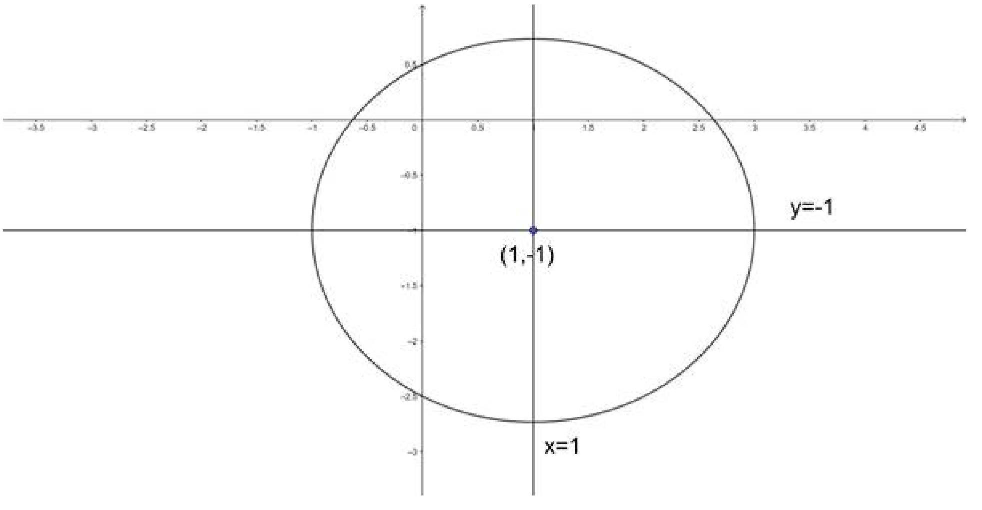
B. $\frac{\sqrt{3}}{2}$

C. $\frac{2}{\sqrt{3}}$

D. none of these

Answer

Given conic is $3x^2 + 4y^2 - 6x + 8y - 5 = 0$. It is re written as



$\Rightarrow 3x^2 + 4y^2 - 6x + 8y - 5 = 0$

$\Rightarrow 3(x^2 - 2x + 1) + 4(y^2 + 2y + 1) = 12$

$\Rightarrow 3(x - 1)^2 + 4(y + 1)^2 = 12$

$\Rightarrow \frac{3(x-1)^2}{12} + \frac{4(y+1)^2}{12} = 1$

$\Rightarrow \frac{(x-1)^2}{4} + \frac{(y+1)^2}{3} = 1$

Comparing with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a^2>b^2)$, The length of the latus - rectum is $\frac{2b^2}{a}$.

$\Rightarrow \frac{2b^2}{a} = \frac{2(3)}{2}$

$\Rightarrow \frac{2b^2}{a} = 3$

∴ The correct option is A

10. Question

The equations of the tangents to the ellipse $9x^2 + 16y^2 = 144$ from the point (2, 3) are

A. y = 3, x = 5

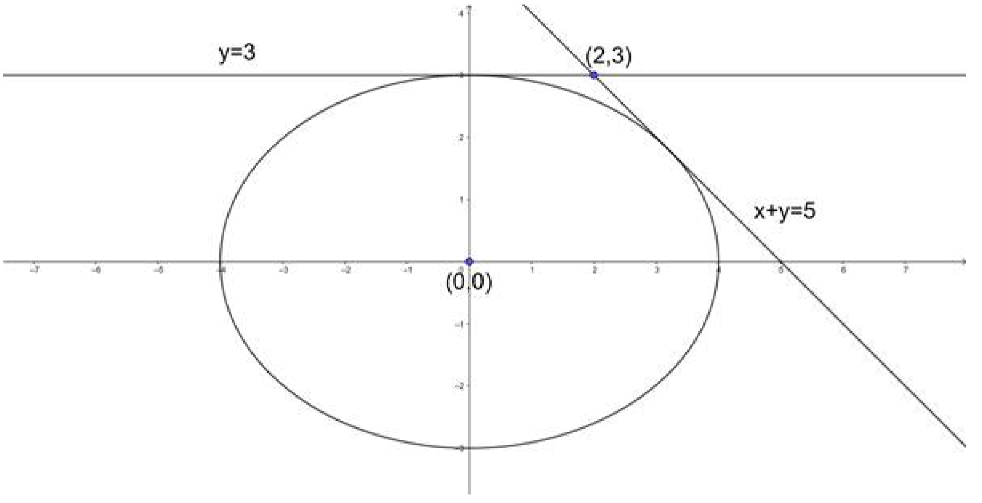
B. x = 2, y = 3

C. x = 3, y = 2

D. x + y = 5, y = 3

Answer

Given that we need to find the equation of the tangents to the ellipse $9x^2 + 16y^2 = 144$ from the point (2,3).



We know that tangent at any point (x_1,y_1) on the ellipse is $S_1 = 0$.

$\Rightarrow S_1 = 0$

$\Rightarrow 9(xx_1) + 16(yy_1) = 144$ .... (1)

This passes through the point (2,3)

$\Rightarrow 9(2x_1) + 16(3y_1) = 144$

$\Rightarrow 18x_1 + 48y_1 = 144$

$\Rightarrow 3x_1 + 8y_1 = 24$

$\Rightarrow 8y_1 = 24 - 3x_1$

$\Rightarrow y_1 = 3 - \frac{3x_1}{8}$ .... - - (2)

Substituting this in the equation of the ellipse we get,

$\Rightarrow 9x_1^2 + 16\left(3 - \frac{3x_1}{8}\right)^2 = 144$

$\Rightarrow 9x_1^2 + 16\left(9 - \frac{9x_1}{4} + \frac{9x_1^2}{64}\right) = 144$

$\Rightarrow 9x_1^2 - 36x_1 + \frac{9x_1^2}{4} = 0$

$\Rightarrow \frac{45x_1^2}{4} - 36x_1 = 0$

$\Rightarrow 9x_1\left(\frac{5x_1}{4} - 4\right) = 0$

$\Rightarrow 9x_1 = 0 \text{ (or) } \frac{5x_1}{4} - 4 = 0$

$\Rightarrow x_1 = 0 \text{ (or) } x_1 = \frac{16}{5}$

From (2)

$\Rightarrow y_1 = 3 - \frac{3(0)}{8}$

$\Rightarrow y_1 = 3$

$\Rightarrow y_1 = 3 - \frac{3\left(\frac{16}{5}\right)}{8}$

$\Rightarrow y_1 = 3 - \frac{6}{5}$

$\Rightarrow y_1 = \frac{9}{5}$

Substituting $x_1 = 0$ and $y_1 = 3$ in (1), we get

$\Rightarrow 9(x(0)) + 16(y(3)) = 144$

$\Rightarrow 48y = 144$

$\Rightarrow y = 3.$

Substituting $x_1 = \frac{16}{5}$ and $y_1 = \frac{9}{5}$ in (1), we get

$\Rightarrow 9\left(x\left(\frac{16}{5}\right)\right) + 16\left(y\left(\frac{9}{5}\right)\right) = 144$

$\Rightarrow \frac{144x}{5} + \frac{144y}{5} = 144$

$\Rightarrow x + y = 5$

∴ The correct option is D

11. Question

The eccentricity of the ellipse $4x^2 + 9y^2 + 8x + 36y + 4 = 0$ is

A. $\frac{5}{6}$

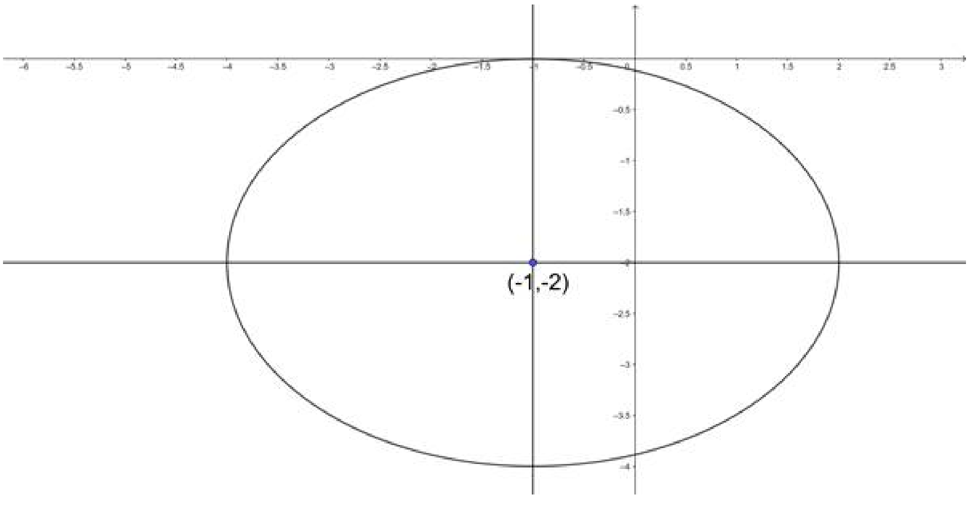
B. $\frac{3}{5}$

C. $\frac{\sqrt{2}}{3}$

D. $\frac{\sqrt{5}}{3}$

Answer

Given that we need to find the eccentricity of the ellipse $4x^2 + 9y^2 + 8x + 36y + 4 = 0$.



$\Rightarrow 4x^2 + 9y^2 + 8x + 36y + 4 = 0$

$\Rightarrow 4(x^2 + 2x + 1) + 9(y^2 + 4y + 4) - 36 = 0$

$\Rightarrow 4(x + 1)^2 + 9(y + 2)^2 = 36$

$$\Rightarrow \frac{4(x+1)^2}{36} + \frac{9(y+2)^2}{36} = 1$$

$$\Rightarrow \frac{(x+1)^2}{9} + \frac{(y+2)^2}{4} = 1$$

Comparing with the standard form $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

Here $a^2 > b^2$

$$\Rightarrow \text{eccentricity}(e) = \sqrt{\frac{a^2-b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{\frac{9-4}{9}}$$

$$\Rightarrow e = \sqrt{\frac{5}{9}}$$

$$\Rightarrow e = \frac{\sqrt{5}}{3}$$

∴ The correct option is D

12. Question

The eccentricity of the ellipse $4x^2 + 9y^2 = 36$ is

A. $\frac{1}{2\sqrt{3}}$

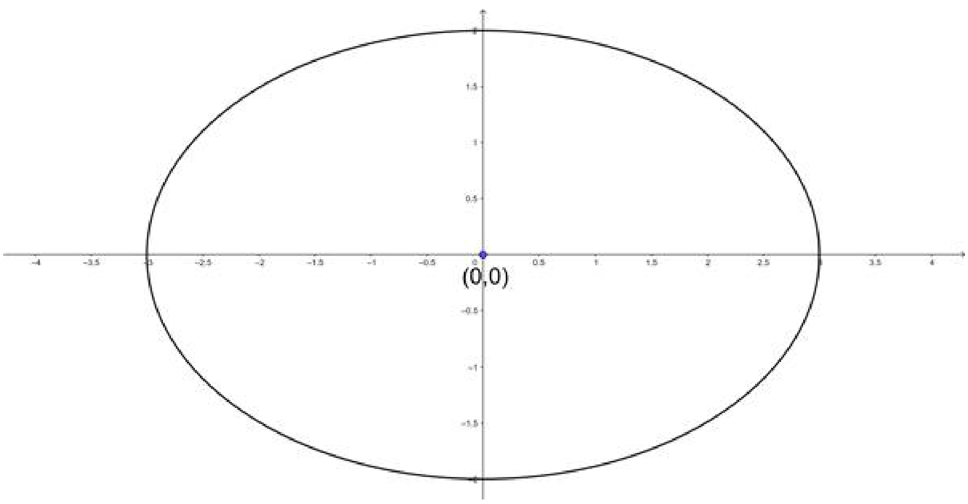
B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{5}}{3}$

D. $\frac{\sqrt{5}}{6}$

Answer

Given that we need to find the eccentricity of the ellipse $4x^2 + 9y^2 = 36$.



$$\Rightarrow 4x^2 + 9y^2 = 36$$

$$\Rightarrow \frac{4x^2}{36} + \frac{9y^2}{36} = 1$$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$

Comparing with the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here $a^2 > b^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{9-4}{9}}$

$\Rightarrow e = \sqrt{\frac{5}{9}}$

$\Rightarrow e = \frac{\sqrt{5}}{3}$

∴ The correct option is C

13. Question

The eccentricity of the ellipse $5x^2 + 9y^2 = 1$ is

A. $\frac{2}{3}$

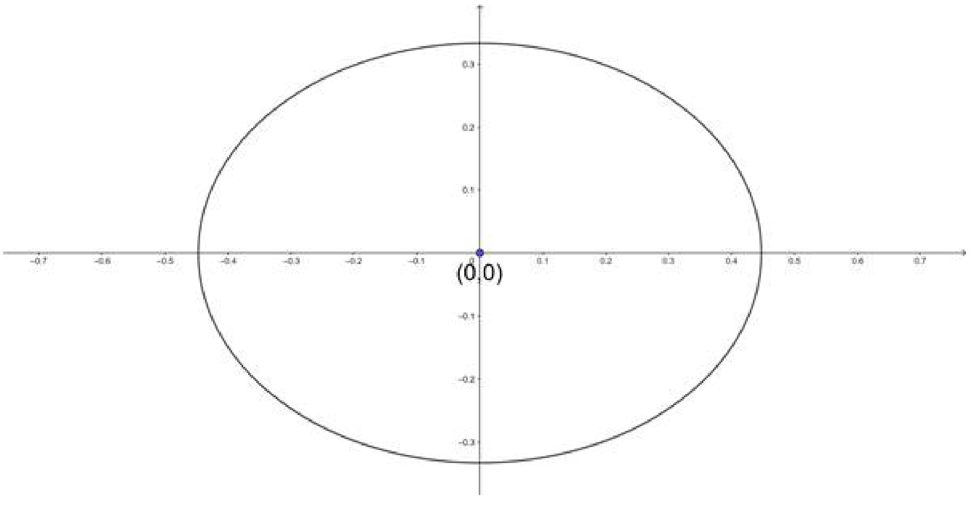
B. $\frac{3}{4}$

C. $\frac{4}{5}$

D. $\frac{1}{2}$

Answer

Given that we need to find the eccentricity of the ellipse $5x^2 + 9y^2 = 1$.



$\Rightarrow 5x^2 + 9y^2 = 1$

$\Rightarrow \frac{x^2}{\frac{1}{5}} + \frac{y^2}{\frac{1}{9}} = 1$

Comparing with the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here $a^2 > b^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{\frac{1}{5}-\frac{1}{9}}{\frac{1}{5}}}$

$\Rightarrow e = \sqrt{\frac{\frac{4}{45}}{\frac{1}{5}}}$

$\Rightarrow e = \sqrt{\frac{4}{9}}$

$\Rightarrow e = \frac{2}{3}$

∴ The correct option is A

14. Question

For the ellipse $x^2 + 4y^2 = 9$

A. The eccentricity is $\frac{1}{2}$

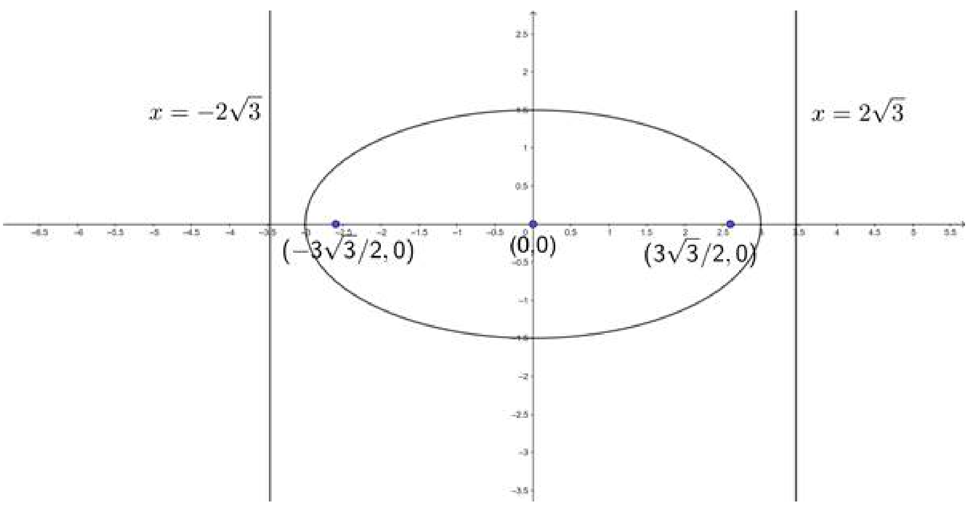
B. The latus rectum is $\frac{3}{2}$

C. A focus is $\left(3\sqrt{3}, 0\right)$

D. A directrix is $x = -2\sqrt{3}$

Answer

Given ellipse is $x^2 + 4y^2 = 9$.



$\Rightarrow x^2 + 4y^2 = 9$

$\Rightarrow \frac{x^2}{9} + \frac{4y^2}{9} = 1$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{\frac{9}{4}} = 1$

Comparing with the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here $a^2>b^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{9-\frac{9}{4}}{9}}$

$\Rightarrow e = \sqrt{\frac{\frac{27}{4}}{9}}$

$\Rightarrow e = \sqrt{\frac{3}{4}}$

$\Rightarrow e = \frac{\sqrt{3}}{2}$

Length of the latus rectum is $\frac{2b^2}{a} = \frac{2\left(\frac{9}{4}\right)}{3} = \frac{3}{2}$

Focus (±ae,0) = $\left(\pm\frac{3\sqrt{3}}{2}, 0\right)$

Directrices are $x = \pm\frac{a}{e}, x = \pm\frac{3}{\frac{\sqrt{3}}{2}}$

Directrices are $x = \pm 2\sqrt{3}$

∴ The correct options are B and D

15. Question

If the latus - rectum of an ellipse is one half of its minor axis, then its eccentricity is

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{\sqrt{3}}{4}$

Answer

Given that we need to find the eccentricity of the ellipse whose latus - rectum is one half of the minor axis.

We know that length of latus rectum is $\frac{2b^2}{a}$ and length of minor axis is 2b.

$\Rightarrow \frac{2b^2}{a} = b$

$\Rightarrow$ a = 2b.

We know that eccentricity of the ellipse is $e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{(2b)^2-b^2}{(2b)^2}}$

$\Rightarrow e = \sqrt{\frac{3b^2}{4b^2}}$

$\Rightarrow e = \frac{\sqrt{3}}{2}$.

∴ The correct option is C

16. Question

An ellipse has its centre at (1, - 1) and semi - major axis = 8 and it passes through the point (1, 3). The equation of the ellipse is

A. $\frac{(x+1)^2}{64}+\frac{(y+1)^2}{16}=1$

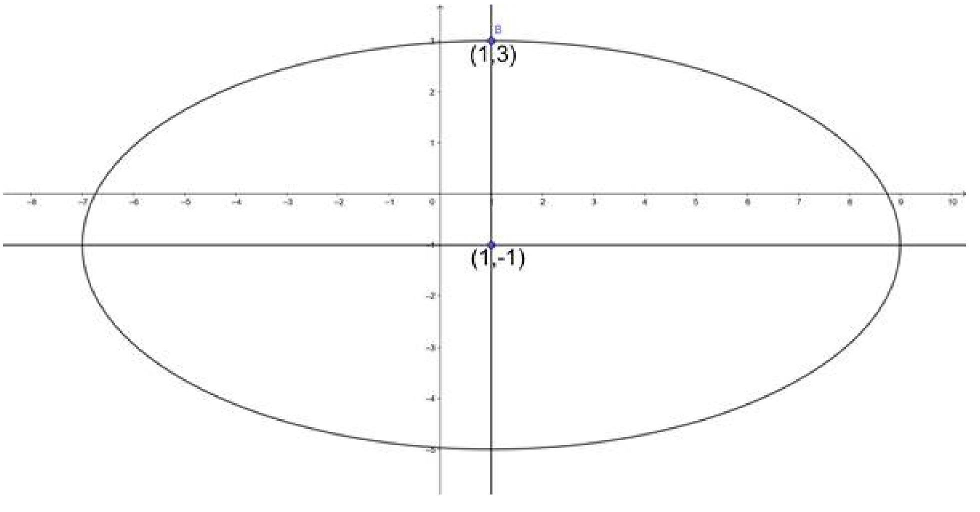
B. $\frac{(x-1)^2}{64}+\frac{(y+1)^2}{16}=1$

C. $\frac{(x-1)^2}{16}+\frac{(y+1)^2}{64}=1$

D. $\frac{(x+1)^2}{64}+\frac{(y-1)^2}{16}=1$

Answer

Given that we need to find the equation of the ellipse whose centre is (1, - 1) and semi - major axis 8 and passes through (1,3).



From the standard equation $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$.

Centre = (p,q) = (1, - 1)

a = 8

a^2 = 64

The equation is $\frac{(x-1)^2}{64} + \frac{(y+1)^2}{b^2} = 1$. This passes through (1,3).

Substituting in the curve, we get,

$\Rightarrow \frac{(1-1)^2}{64} + \frac{(3+1)^2}{b^2} = 1$

$\Rightarrow \frac{0}{64} + \frac{16}{b^2} = 1$

$\Rightarrow b^2 = 16$

The equation of the ellipse is $\frac{(x-1)^2}{64} + \frac{(y+1)^2}{16} = 1$.

∴ The correct option is B

17. Question

The sum of the focal distances of any point on the ellipse $9x^2 + 16y^2 = 144$ is

A. 32

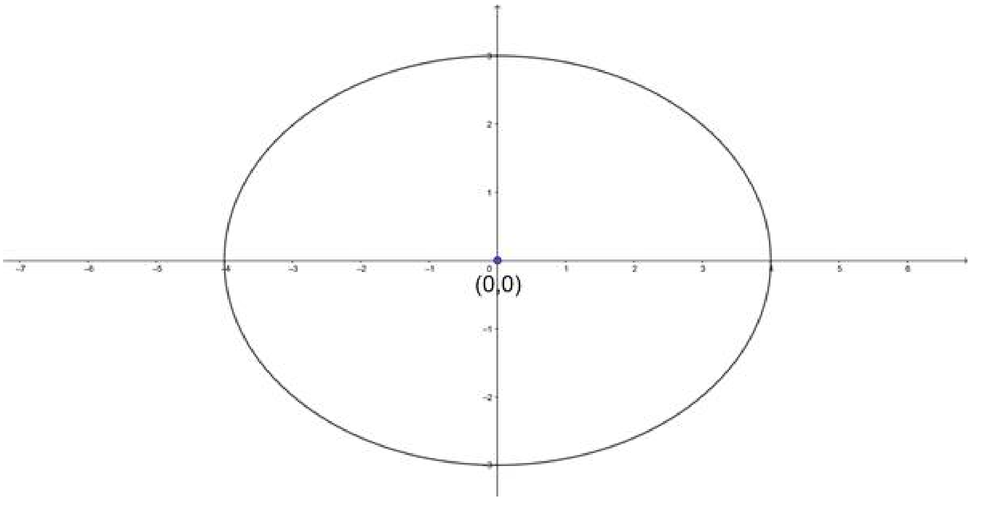
B. 18

C. 16

D. 8

Answer

Given ellipse is $9x^2 + 16y^2 = 144$. It is rewritten as



$\Rightarrow \frac{9x^2}{144} + \frac{16y^2}{144} = 1$

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$

Comparing with standard equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ we get

$\Rightarrow a^2 = 16$

$\Rightarrow a = 4$

We know that the sum of the focal distances of any point on the ellipse is 2a.

$\Rightarrow 2(4) = 8$

∴ The correct option is D

18. Question

If (2, 4) and (10, 10) are the ends of a latus - rectum of an ellipse with eccentricity 1/2, then the length of semi - major axis is

A. $\frac{20}{3}$

B. $\frac{15}{3}$

C. $\frac{40}{3}$

D. none of these

Answer

Given (2,4) and (10,10) are the ends of the latus - rectum and eccentricity is $\frac{1}{2}$.

We know that length of the latus rectum is $\frac{2b^2}{a}$.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow \frac{2b^2}{a} = \sqrt{(2-10)^2 + (4-10)^2}$

$\Rightarrow \frac{2b^2}{a} = \sqrt{(-8)^2 + (-6)^2}$

$\Rightarrow \frac{2b^2}{a} = \sqrt{64 + 36}$

$\Rightarrow \frac{2b^2}{a} = \sqrt{100}$

$\Rightarrow 2b^2 = 10a$

$\Rightarrow b^2 = 5a$

We know that $b^2 = a^2(1 - e^2)$

$\Rightarrow a^2(1 - e^2) = 5a$

$\Rightarrow a\left(1-\left(\frac{1}{2}\right)^2\right) = 5$

$\Rightarrow a = \frac{5}{1-\frac{1}{4}}$

$\Rightarrow a = \frac{5}{\frac{3}{4}}$

$\Rightarrow a = \frac{20}{3}$

∴ The correct option is A

19. Question

The equation $\frac{x^2}{2-\lambda} + \frac{y^2}{\lambda - 5} + 1 = 0$ represents an ellipse, if

A. λ<5

B. λ <2

C. 2<λ<5

D. λ<2 and λ>5

Answer

Given equation is $\frac{x^2}{2-\lambda} + \frac{y^2}{\lambda-5} + 1 = 0$

$\Rightarrow \frac{x^2}{2-\lambda} + \frac{y^2}{\lambda-5} = -1$

$\Rightarrow \frac{x^2}{\lambda-2} + \frac{y^2}{5-\lambda} = 1$

Comparing with standard equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ we get

$\Rightarrow \lambda - 2>0$

$\Rightarrow \lambda>2$ .... - (1)

$\Rightarrow 5 - \lambda>0$

$\Rightarrow \lambda<5$ .... - (2)

From (1) and (2),

$\Rightarrow 2<\lambda<5$

∴ The correct option is C

20. Question

The eccentricity of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$, is

A. $\frac{25}{16}$

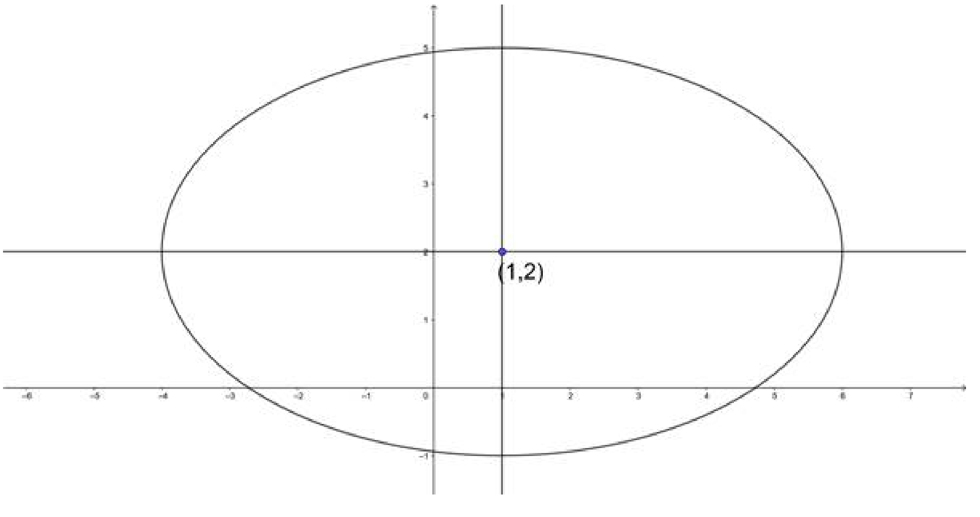
B. $\frac{4}{5}$

C. $\frac{16}{25}$

D. $\frac{5}{4}$

Answer

Given that we need to find the eccentricity of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$.



$\Rightarrow 9x^2 + 25y^2 - 18x - 100y - 116 = 0$

$\Rightarrow 9(x^2 - 2x + 1) + 25(y^2 - 4y + 4) - 225 = 0$

$\Rightarrow 9(x - 1)^2 + 25(y - 2)^2 = 225$

$\Rightarrow \frac{9(x-1)^2}{225} + \frac{25(y-2)^2}{225} = 1$

$\Rightarrow \frac{(x-1)^2}{25} + \frac{(y-2)^2}{9} = 1$

Comparing with the standard form $\frac{(x-p)^2}{a^2}+\frac{(y-q)^2}{b^2}=1$

Here $a^2>b^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e=\sqrt{\frac{25-9}{25}}$

$\Rightarrow e=\sqrt{\frac{16}{25}}$

$\Rightarrow e=\frac{4}{5}$

∴ The correct option is B

21. Question

If the major axis of an ellipse is three times the minor axis, then its eccentricity is equal to

A. $\frac{1}{3}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{2\sqrt{2}}{3}$

E. $\frac{2}{3\sqrt{2}}$

Answer

Given that the major axis of an ellipse is three times the minor axis.

$\Rightarrow$ a = 3b

We know that eccentricity(e) = $\sqrt{\frac{a^2-b^2}{a^2}}$

$\Rightarrow e=\sqrt{\frac{(3b)^2-b^2}{(3b)^2}}$

$\Rightarrow e=\sqrt{\frac{8b^2}{9b^2}}$

$\Rightarrow e=\sqrt{\frac{8}{9}}$

$\Rightarrow e=\frac{2\sqrt{2}}{3}$

∴ The correct option is D

22. Question

The eccentricity of the ellipse $25x^2 + 16y^2 = 400$ is

A. $\frac{3}{5}$

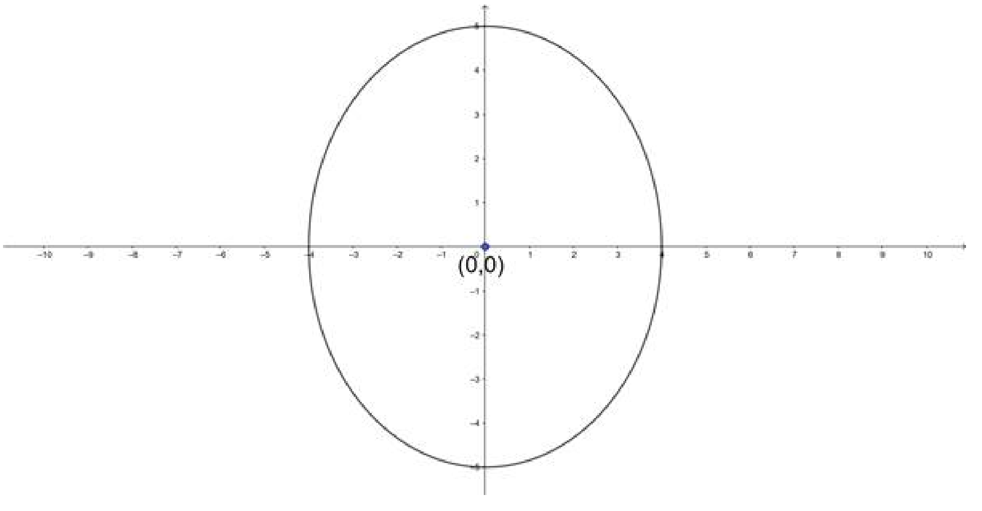
B. $\frac{1}{3}$

C. $\frac{2}{5}$

D. $\frac{1}{5}$

Answer

Given that we need to find the eccentricity of the ellipse $25x^2 + 16y^2 = 400$.



$\Rightarrow 25x^2 + 16y^2 = 400$

$\Rightarrow \frac{25x^2}{400} + \frac{16y^2}{400} = 1$

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1$

Comparing with the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here $b^2 > a^2$

$\Rightarrow$ eccentricity(e) = $\sqrt{\frac{b^2-a^2}{b^2}}$

$\Rightarrow e = \sqrt{\frac{25-16}{25}}$

$\Rightarrow e = \sqrt{\frac{9}{25}}$

$\Rightarrow e = \frac{3}{5}$

∴ The correct option is A

23. Question

The eccentricity of the ellipse $5x^2 + 9y^2 = 1$ is

A. $\frac{2}{3}$

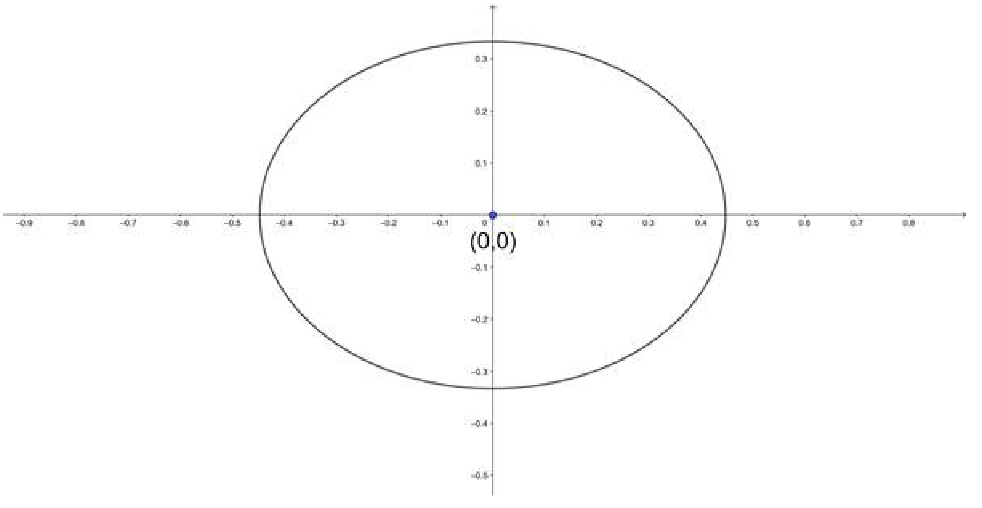
B. $\frac{3}{4}$

C. $\frac{4}{5}$

D. $\frac{1}{2}$

Answer

Given that we need to find the eccentricity of the ellipse $5x^2 + 9y^2 = 1$.



$\Rightarrow 5x^2 + 9y^2 = 1$

$\Rightarrow \frac{x^2}{\frac{1}{5}} + \frac{y^2}{\frac{1}{9}} = 1$

Comparing with the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here $a^2 > b^2$

$\Rightarrow$ eccentricity(e) $= \sqrt{\frac{a^2 - b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{\frac{1}{5} - \frac{1}{9}}{\frac{1}{5}}}$

$\Rightarrow e = \sqrt{\frac{\frac{4}{45}}{\frac{1}{5}}}$

$\Rightarrow e = \sqrt{\frac{4}{9}}$

$\Rightarrow e = \frac{2}{3}$

∴ The correct option is A

24. Question

The eccentricity of the ellipse $4x^2 + 9y^2 = 36$ is

A. $\frac{1}{2\sqrt{3}}$

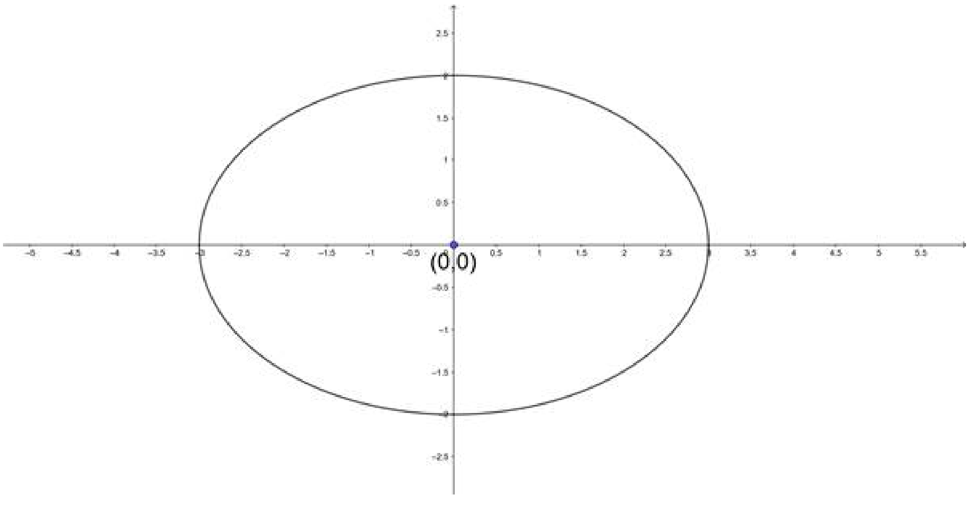
B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{5}}{3}$

D. $\frac{\sqrt{5}}{6}$

Answer

Given that we need to find the eccentricity of the ellipse $4x^2 + 9y^2 = 36$.



$\Rightarrow 4x^2 + 9y^2 = 36$

$\Rightarrow \frac{4x^2}{36} + \frac{9y^2}{36} = 1$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$

Comparing with the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here $a^2 > b^2$

$\Rightarrow$ eccentricity(e) $= \sqrt{\frac{a^2 - b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{9-4}{9}}$

$\Rightarrow e = \sqrt{\frac{5}{9}}$

$\Rightarrow e = \frac{\sqrt{5}}{3}$

∴ The correct option is C